A modern look at short antennas for military and amateur radio applications

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Optimizing Rod Antennas for Manpack Systems for Both Amateur and Military Applications

Rod antennas, widely used in portable radio systems, are crucial for reliable communication in amateur and military contexts. These systems must perform well under varying environmental and ionospheric conditions, where signal propagation is frequency-dependent. At any time of day, an optimal frequency or small band of frequencies between two points remains stable for several hours. We can perform best by properly matching an antenna to this frequency. In this discussion, we will introduce a combination of matching techniques that improve transmission and provide better coverage than previous methods.

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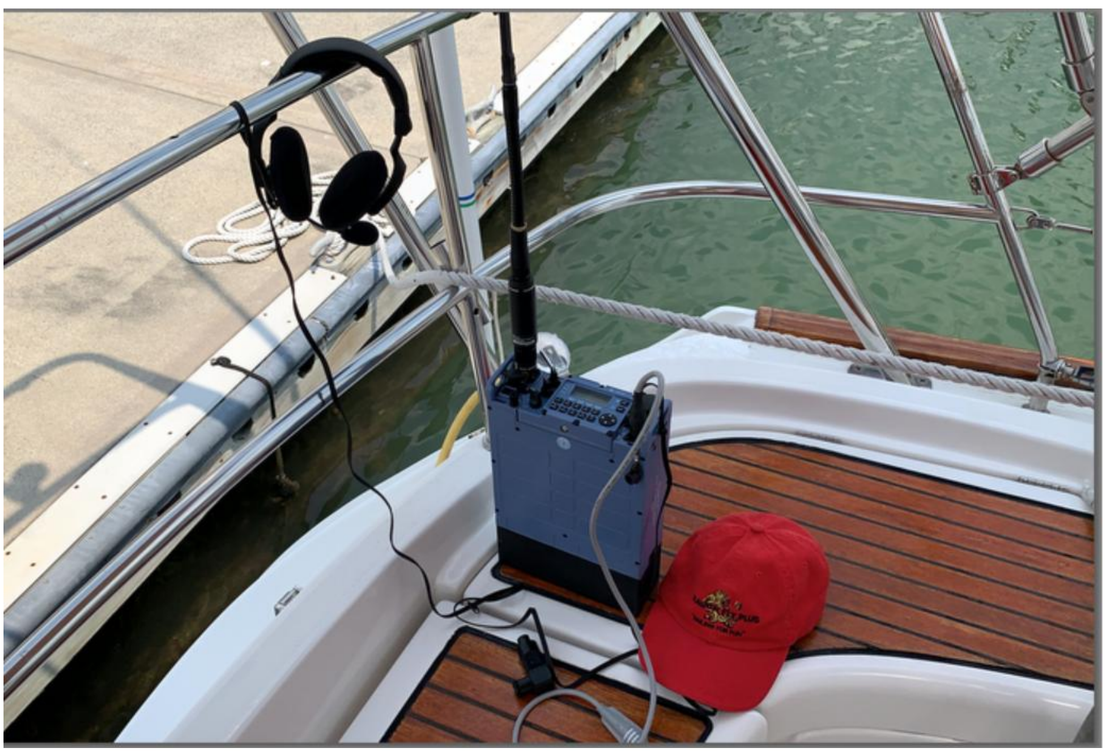
Typical applications of portable radios in the HF frequency range are either in the field, in parks or as shown on the water. The R&S®M3TR 1.5 to 108 MHz manpack with 20 Watt output is capable of all modulation types and is frequently called multiband, multimode, multirole. One challenge in today's military missions occurs in joint operations between multinational armed forces. Interoperability of the equipment, especially in the field of communications, is therefore the primary objective of the international partners responsible for creating one of the most important aspects for efficient cooperation. The R&S®M3TR features maximum flexibility in terms of frequency bands and waveforms for practically all services and platforms. In the photo, I am using it with a 3 m vertical rod. Perfect grounding is achieved by connecting it to the boat's railing.

The purpose of this presentation is to show the performance of the antenna with a base loading coil and the matching circuit needed, and then later the significant advantage of using an almost top-mounted loading coil between two rods, which are now properly modeled as transmission lines.

The published radiation resistance value of typically 40 ohms is wrong as the needed integral equation is not properly handled; the correct solution is shown here. Finally, in contradiction to practically all publications, there is still radiating current at the end of the rod antenna.

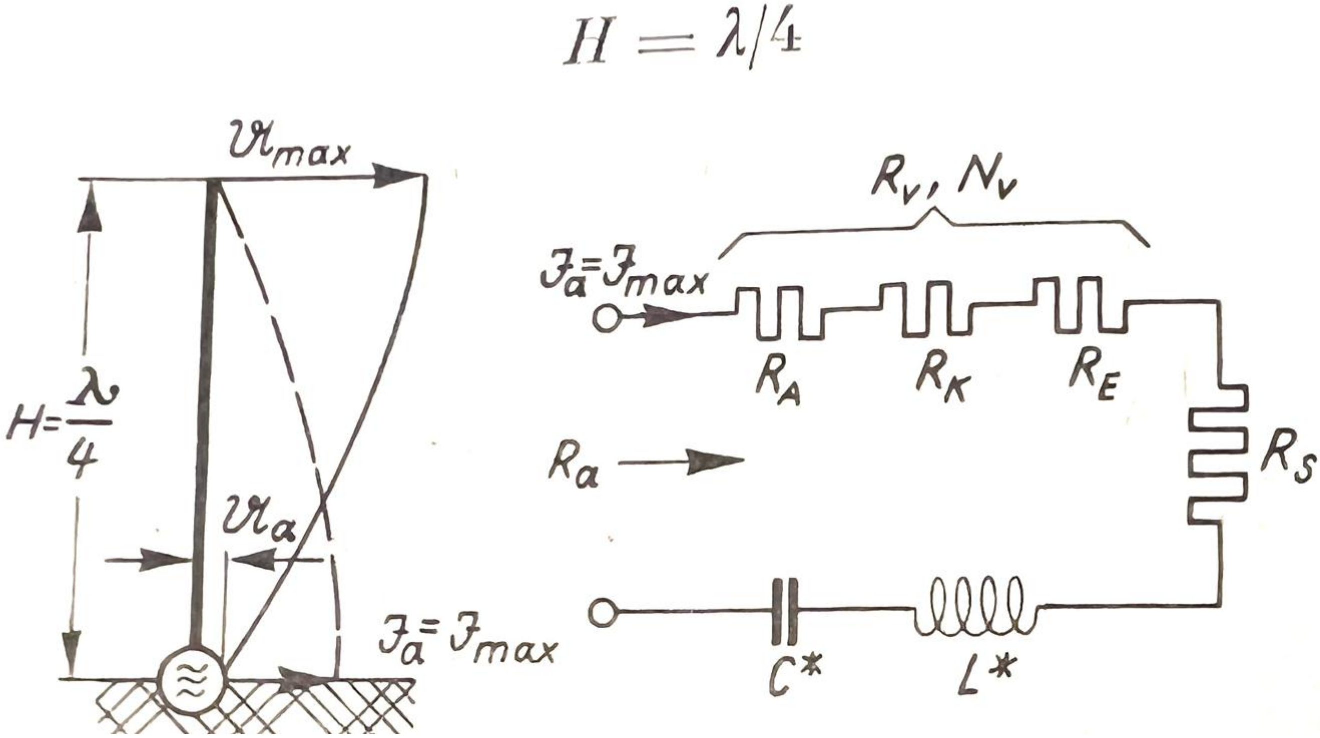


Thera are many applications for HF manpacks on land and on sea. They have a typical output power of 20 W. Modern versions cover 1.5 to 108 MHz, while the antenna tuner covers 1.5 to 30 MHz (max. 54 MHz). Part of the presentation was taken from the forthcoming book by:

Raghunath K. Shevgaonkar and Ulrich L. Rohde

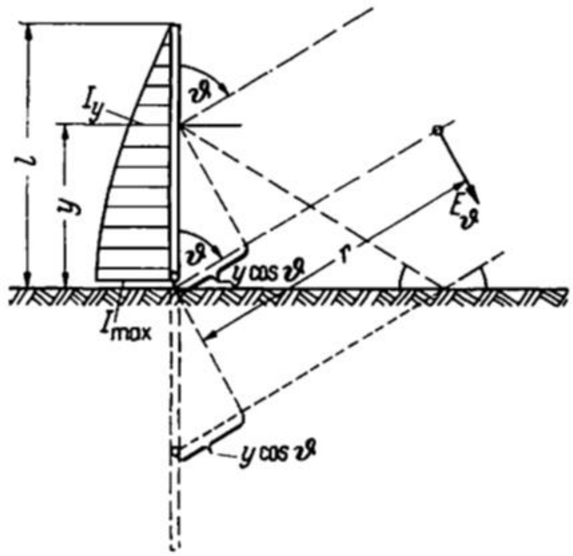
Springer

Quarter wave antenna:



Now to the radiation:

The radiation can be calculated as follows:



In the treatment of the Hertzian dipole, we base our calculations on a location-independent current because $\Delta l \ll \lambda$. If we now consider an arbitrarily long antenna, we must take into account the current's dependency on the location. Analogous to the transmission line, we again assume a sinusoidal current distribution over the length. Based on this approximation which is valid for thin antennas, we would like to calculate the vertical radiation pattern for a vertical antenna over ground. We imagine that the antenna in Fig. 6.2/1 is constructed from a large number of Hertzian dipoles. For the far field ($r \gg \lambda$ and $r \gg l$), we obtain the following as the contribution of the current element $I_y \mathrm{d}y$ taking into account the (ideally reflective) ground:

$$\mathrm{d}E_\vartheta = \mathrm{j}Z_0 \frac{I_\mathrm{y}\,\mathrm{d}y}{2\lambda} \sin\vartheta \left[\frac{\mathrm{e}^{-\mathrm{j}\frac{2\pi}{\lambda}(r - \mathrm{y}\cos\vartheta)}}{r - y\cos\vartheta} + \frac{\mathrm{e}^{-\mathrm{j}\frac{2\pi}{\lambda}(r + \mathrm{y}\cos\vartheta)}}{r + \mathrm{y}\cos\vartheta} \right] \qquad (6.2/1)$$

Here, we have

$$I_y = I_\mathrm{max} \sin\frac{2\pi(l - y)}{\lambda} \qquad (6.2/2)$$

In equation (6.2/1), we consider only the phase difference and not the amplitude difference between the two components. We thus obtain

$$\mathrm{d}E_\vartheta = \quad \mathrm{j}Z_0 I_{\max}\sin\frac{2\pi(l-y)}{\lambda}\frac{\mathrm{d}y}{\lambda}\sin\vartheta\frac{\mathrm{e}^{-\mathrm{j}\frac{2\pi r}{\lambda}}}{r}\frac{1}{2}\left[\mathrm{e}^{\mathrm{j}\frac{2\pi y}{\lambda}\cos\vartheta}+\mathrm{e}^{-\mathrm{j}\frac{2\pi y}{\lambda}\cos\vartheta}\right]$$

$$\mathrm{d}E_\vartheta = \quad \mathrm{j}\cdot 60\,\Omega\;\cdot I_{\max}\frac{\mathrm{e}^{-\mathrm{j}\frac{2\pi r}{\lambda}}}{r}\sin\vartheta\left\{\sin\frac{2\pi l}{\lambda}\cos\frac{2\pi y}{\lambda}-\cos\frac{2\pi l}{\lambda}\sin\frac{2\pi y}{\lambda}\right\} \times\cos\left(\frac{2\pi y}{\lambda}\cos\vartheta\right)\mathrm{d}\frac{2\pi y}{\lambda},$$

$$E_\vartheta = \quad \mathrm{j}\cdot 60\,\Omega\;\cdot I_{\max}\frac{\mathrm{e}^{-\mathrm{j}\frac{2\pi r}{\lambda}}}{r}\sin\vartheta\left\{\sin\frac{2\pi l}{\lambda}\int_{u=0}^{\frac{2\pi l}{\lambda}}\cos u\cos(u\cos\vartheta)\mathrm{d}u -\cos\frac{2\pi l}{\lambda}\int_{u=0}^{\frac{2\pi l}{\lambda}}\sin u\,\cos(u\cos\vartheta)\mathrm{d}u\right\}$$

For $u = 2\pi y/\lambda$, integration leads to

$$E_\vartheta = \mathrm{j}\cdot 60\,\Omega\cdot I_{\max}\frac{\mathrm{e}^{-\mathrm{j}\frac{2\pi r}{\lambda}}}{\mathrm{r}\sin\vartheta}\left\{\sin\frac{2\pi l}{\lambda}\left[\sin\frac{2\pi l}{\lambda}\cos\left(\frac{2\pi l}{\lambda}\cos\vartheta\right)-\cos\vartheta\cos\frac{2\pi l}{\lambda}\sin\left(\frac{2\pi l}{\lambda}\cos\vartheta\right)\right] + \cos\frac{2\pi l}{\lambda}\left[\cos\frac{2\pi l}{\lambda}\cos\left(\frac{2\pi l}{\lambda}\cos\vartheta\right)+\cos\vartheta\sin\frac{2\pi l}{\lambda}\sin\left(\frac{2\pi l}{\lambda}\cos\vartheta\right)-1\right]\right\}$$

After multiplying the values in brackets [], we can simplify as follows:

$$E_\vartheta = \mathrm{j}\cdot 60\,\Omega\cdot I_{\max}\frac{\mathrm{e}^{-\mathrm{j}\frac{2\pi r}{\lambda}}}{r}\frac{\cos\left(\frac{2\pi l}{\lambda}\cos\vartheta\right)-\cos\frac{2\pi l}{\lambda}}{\sin\vartheta} = E_{\delta_\mathrm{r}}\cdot F\left(\vartheta;\frac{l}{\lambda}\right) \tag{6.2/3}$$

The function

$$F\left(\vartheta;\frac{l}{\lambda}\right) = \frac{\cos\left(\frac{2\pi l}{\lambda}\cos\vartheta\right)-\cos\frac{2\pi l}{\lambda}}{\sin\vartheta} \tag{6.2/4}$$

provides the vertical pattern of the antenna.

For $\cos\vartheta = 0$, the field strength for $\vartheta = \pi/2$ has the magnitude

$$|E_\vartheta|_{\vartheta=\pi/2} = \frac{60\,\Omega\;\cdot I_{\max}}{r}\left(1-\cos\frac{2\pi l}{\lambda}\right) \tag{6.2/5}$$

Due to the symmetry, the radiation pattern is independent of the azimuth angle φ. The horizontal radiation pattern is thus circular.

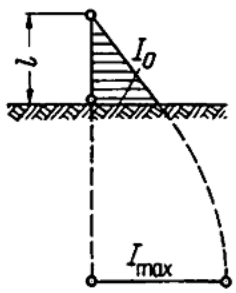


Fig. 6.2/2. Current distribution for a short antenna

We will now consider the case of an electrically short antenna over ground and calculate the field strength and radiation resistance. For short antennas ($l \leq \lambda/8$), we can use the approximations $\cos x \approx 1 - x^2/2$ and $\sin x \approx x$ instead of equation (6.2/3) to obtain a simpler relationship for the electric field strength:

$$E_\vartheta = \mathrm{j}\frac{2\pi \cdot 60\,\Omega}{r} I_{max} \frac{\pi l^2}{\lambda^2} \sin\vartheta \, \mathrm{e}^{-\mathrm{j}\frac{2\pi r}{\lambda}} \qquad (6.2/6)$$

If $l/\lambda \ll 1$ (Fig. 6.2/2), we have

$$I_0 = I_{max} \sin\frac{2\pi l}{\lambda} \approx I_{max} \frac{2\pi l}{\lambda}$$

and it then follows that

$$E_\vartheta = \mathrm{j}Z_0 \frac{I_0 l}{2\lambda} \frac{\sin\vartheta}{r} \mathrm{e}^{-\mathrm{j}\frac{2\pi r}{\lambda}} \qquad (6.2/7)$$

Unlike the case of the Hertzian dipole, here the current drops linearly to 0; the field strength of the short antenna is thus half that of the Hertzian dipole.

Using the procedure described in section 6.1.5, we can now calculate the radiation resistance of the electrically short antenna from E_ϑ:

$$
\begin{aligned}
P_{\text{rad}} &= \int_0^{\pi/2} \frac{E_\vartheta^2}{Z_0} \cdot 2\pi r^2 \sin\vartheta \, \mathrm{d}\vartheta \\
&= Z_0 \left(\frac{I_0 l}{2\lambda}\right)^2 \cdot 2\pi \int_0^{\frac{\pi}{2}} \sin^3\vartheta \, \mathrm{d}\vartheta \qquad (6.2/8) \\
R_{\text{rad}} &= 60\pi^2 \, \Omega \left(\frac{l}{\lambda}\right)^2 \int_0^{\pi/2} \sin^3\vartheta \, \mathrm{d}\vartheta \\
R_{\text{rad}} &= 40\pi^2 \, \Omega \left(\frac{l}{\lambda}\right)^2 = 395 \, \Omega \left(\frac{l}{\lambda}\right)^2 \qquad (6.2/9)
\end{aligned}
$$

Example: $\lambda = 30\ \mathrm{m}\ (f = 10\ \mathrm{MHz})$; $l = 7\ \mathrm{m}$ (shortwave antenna). Thus, $\frac{l}{\lambda} = 0.233$ and $R_{\text{rad}} = 21.5\ \Omega$. The radiation resistance of this short antenna is relatively low.

Now that we have examined the properties of electrically short antennas where $l \leq \lambda/8$, we will turn our attention to longer antennas. Like for the transmission line, we will first consider the case in which $l = \lambda/4$. For the vertical radiation pattern of the $\lambda/4$ antenna over ground, we obtain the following from equation (6.2/4):

$$F\left(\vartheta; \frac{l}{\lambda} = \frac{1}{4}\right) = \frac{\cos\left(\frac{\pi}{2}\cos\vartheta\right)}{\sin\vartheta} \qquad (6.2/19)$$

We can determine the radiated power and the radiation resistance of the $\lambda/4$ antenna over ground based on the power radiated in the half sphere:

$$P_{\text{rad}} = \tilde{I}_0^2 R_{\text{rad}} = \int_{\vartheta=0}^{\vartheta=\pi/2} \frac{\bar{E}_\vartheta^2}{\mathrm{Z}_0} \cdot 2\pi r^2 \sin\vartheta \, \mathrm{d}\vartheta = \tilde{I}_0^2 \cdot 60\ \Omega \int_0^{\pi/2} F^2\left(\vartheta, \frac{l}{\lambda}\right) \sin\vartheta \, \mathrm{d}\vartheta$$

$$F^2 = \int_0^{\pi/2} \frac{\cos^2(\pi/2 \cos\theta)}{\sin^2\theta} \cdot \sin\theta d\theta$$

$$F^2 = \int_0^{\pi/2} \frac{\cos^2(\pi/2 \cos\theta)}{\sin\theta} \cdot d\theta$$

Now, setting $u = \cos\theta$, we have

$$
\begin{aligned}
&du = -\sin\theta \cdot d\theta \Rightarrow d\theta = \left(-\frac{du}{\sin\theta}\right) \\
&\sin^2\theta + \cos^2\theta = 1 \\
&\sin^2\theta + u^2 = 1 \\
&\sin\theta = \sqrt{1 - u^2}
\end{aligned}
$$

Limits: If $\theta = 0$, then $u = 1$ and if $\theta = \pi/2$, then $u = 0$.

Substituting all the values into the equation, we obtain:

$$
\begin{aligned}
F \quad &= \int_1^0 \frac{\cos^2(\frac{\pi}{2}\cdot u)}{\sin\theta} . \frac{-du}{\sin\theta} \\
&= \int_0^1 \frac{\cos^2(\frac{\pi}{2}\cdot u)}{1-u^2} . du
\end{aligned}
$$

This equation does not have a simple closed form because the upper limit goes to ∞. However, we can find the solution using a numerical method. We will find the solution for F using four points.

Substituting a value of four into the equation, we obtain

$$
F = \int_0^{\pi/2} \frac{\cos^2(\frac{\pi}{2}\cdot\cos\theta)}{\sin\theta} d\theta
$$

$$
\int_a^b f(c)dx \approx \frac{b-a}{n}\left(\frac{f(a)}{2} + \sum_{k=1}^{n-1}\left(f(a) + k\frac{b-a}{n}\right)\right)
$$

where

$$
\begin{aligned}
a &= 0 \qquad n = 4 \text{ points} \\
b &= \pi/2 \quad k = 1,2,3
\end{aligned}
$$

The points of the function are as follows:

$$
f(\theta)|_{\theta=0} = f(0) = \frac{\cos^2\left(\frac{\pi}{2}\cdot\cos 0\right)}{\sin 0} = 0
$$

$$
f(\theta)|_{\theta=\frac{\pi}{8}} = f(\frac{\pi}{8}) = \frac{\cos^2\left(\frac{\pi}{2}\cdot\cos\frac{\pi}{8}\right)}{\sin\frac{\pi}{8}} = 0.0368
$$

$$
f(\theta)|_{\theta=\frac{\pi}{4}} = f(\frac{\pi}{4}) = \frac{\cos^2\left(\frac{\pi}{2}\cdot\cos\frac{\pi}{4}\right)}{\sin\frac{\pi}{4}} = 0.2838
$$

$$
f(\theta)|_{\theta=\frac{3\pi}{8}} = f(\frac{3\pi}{8}) = \frac{\cos^2\left(\frac{\pi}{2}\cdot\cos\frac{3\pi}{8}\right)}{\sin\frac{3\pi}{8}} = 0.7349
$$

$$
f(\theta)|_{\theta=\frac{\pi}{2}} = f(\frac{\pi}{2}) = \frac{\cos^2(\frac{\pi}{2}.\cos\frac{\pi}{2})}{\sin\frac{\pi}{2}} = 1
$$

$$P_{\text{rad}} = 60\ \Omega \left[\frac{\pi}{8}\left(0 + 0.0368 + 0.28838 + 0.7349 + \frac{1}{2}\right)\right]$$
$$P_{\text{rad}} = \tilde{I}_0^2 \cdot 60\ \Omega \cdot \frac{\pi}{8}(1.555)$$
$$P_{\text{rad}} = \tilde{I}_0^2 \cdot 60\ \Omega \cdot 0.6105$$
$$P_{\text{rad}} = 36.63\ \Omega \cdot \tilde{I}_0^2$$

$$R_{\text{rad}} = 36.63\ \Omega$$

We will now perform the analogous calculations for the $\lambda/2$ antenna. We can obtain its vertical pattern:

$$F\left(\vartheta; \frac{l}{\lambda} = \frac{1}{2}\right) = \frac{\cos(\pi \cos\vartheta) + 1}{\sin\vartheta} = \frac{2\cos^2\left(\frac{\pi}{2}\cos\vartheta\right)}{\sin\vartheta}$$

We obtain the radiation resistance using the same approach as above:

$$P_{\text{rad}} = \tilde{I}_0^2 R_{\text{rad}} = \tilde{I}_0^2 \cdot 60\ \Omega \int_0^{\pi/2} \frac{[\cos(\pi\cos\vartheta) + 1]^2}{\sin^2\vartheta} \sin\vartheta\ \mathrm{d}\vartheta$$

$$R_{\text{rad}} = 60\ \Omega \cdot 1.66 = 99.5\ \Omega \approx 100\ \Omega$$

Note that this radiation resistance is referred to the current maximum while the feed point (base) is located at the voltage maximum. The base resistance R_F differs similarly from R_S like the input and termination impedance of a $\lambda/4$ line:

$$R_{\text{F}} = \frac{Z_{\text{a}}^2}{R_{\text{rad}}}$$

$Z_a^2 = L_A/C_A$ is the line characteristic impedance of the antenna with respect to ground:

$$Z_{\text{a}} \approx 60\ \Omega \left(\ln\frac{2l}{d} - 0.6\right)$$

$2l/d$ is the antenna's slenderness ratio ($d =$ diameter of antenna conductor).

In our discussion so far, we assumed a sinusoidal current distribution on the antenna. This is acceptable only for slender antennas for which l/d is large. In reality, the current distribution is not perfectly sinusoidal because the antenna behaves like a highly attenuated line. The attenuation is primarily a consequence of the radiated power and can be taken into account by assuming that the radiation resistance is distributed over the entire antenna length.

The equivalent circuit of the half wave antenna ($R_s = 99\ \Omega$) at the voltage resonance point is a trap circuit with the input resistance

$$R_{\text{F}} = \frac{Z_{\text{a}}^2}{R_{\text{rad}(\lambda/2)}} \approx \frac{Z_{\text{a}}^2}{100\ \Omega}$$

Here, Z_a is the antenna's characteristic impedance and R_S is its radiation resistance. Since the reactance of a $\lambda/2$ dipole corresponds to that of a parallel resonant circuit, we can compensate the frequency response of the input impedance in a narrow frequency range by connecting a series resonant circuit to the input (e.g. an open $\lambda/4$ line). Treating the antennas like lines, we ascertain that the quantities per unit length L' and C' are location-dependent. This is because thick antennas exhibit voltage resonance at lengths less than $\lambda/2$.

Ground resistance. Antenna efficiency. We examined the influence of the ground reflection in section 6.1.5. Fig. 6.2/3a illustrates the behavior of the electric field lines for a vertical antenna over ground. These field lines penetrate the surface of the earth and produce a current that flows back to the ground point. Current heat losses arise in this manner. For radiation resistance R_S and total effective loss resistance R_v, we obtain the antenna efficiency

$$\eta = \frac{R_s}{R_s + R_v} \qquad (6.2/10)$$

For electrically short antennas with low radiation resistance values of only a few ohms (especially long-wave and extremely long-wave systems), the resulting antenna efficiency can be very low. We can reduce R_v in such cases with a ground network (Fig. 6.2/3b) or – especially in case of unfavorable ground conditions – a wire network extending over the ground as a “counterpoise”.

All antennas that are not excited vs. ground (e.g. dipoles for shortwave or VHF) benefit from significant independence from the ground resistance as long as the antenna is mounted on a tower, for example.

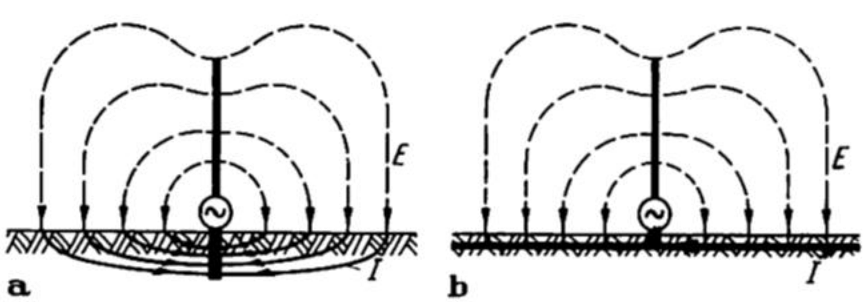


Fig. 6.2/3a,b. a. Ground currents in the vicinity of the antenna’s base result in losses; b. A ground network provides help

6.2.2.3 Effective height of electrically short antennas. The open-circuit voltage V_0 of an antenna is proportional to the antenna field strength E where the antenna is located:

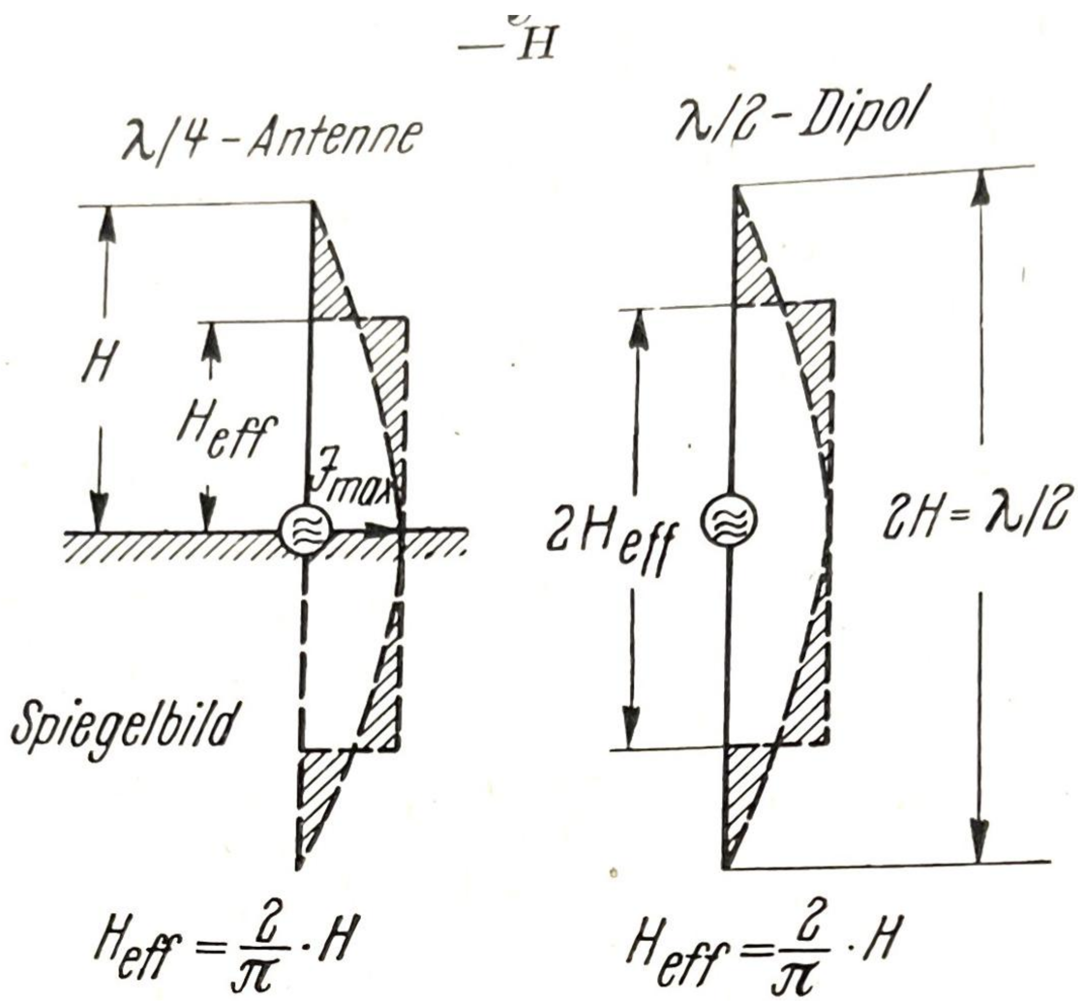
$$U_0 = h_{\text{eff}} E \,. \tag{6.2/11}$$

The proportionality factor h_{eff} has the dimension of length and is known as the "effective height". If the current in the antenna is location-independent (Hertzian dipole), then h_{eff} corresponds to the antenna's geometrical length l. Otherwise, the effective height is less due to the non-uniform current distribution. In this general case, we obtain h_{eff} by converting the current area into a rectangle with the same area having the maximum current I_0 as its base as shown in Fig. 6.2/4. Its height is then equal to h_{eff}. Computationally, we have

$$I_0 h_{\text{eff}} = \int_0^h I_y \, \mathrm{d}y \,, \qquad h_{\text{eff}} = \int_0^h \frac{I_y}{I_0} \, \mathrm{d}y \,. \tag{6.2/12}$$

The definition of the effective height is closely related to that of the effective area A (see section 6.1.7):

$$A = \frac{h_{\text{eff}}^2}{4} \frac{Z_0}{R_s} \,, \qquad h_{\text{eff}} = 2\sqrt{A \frac{R_s}{Z_0}} \,. \tag{6.2/13}$$



The radiated power is

$$N_s = \frac{\pi}{3} \cdot \frac{Z_0}{\lambda^2} \cdot \left[I_{\text{max , eff}} \cdot \frac{2}{\pi} \cdot 2H \right]^2 = \frac{\pi}{3} \cdot Z_0 \cdot \left[\frac{2H_{\text{eff}}}{\lambda} \right]^2 \cdot I^2_{\text{max , eff}} = R_s \left(I_{\text{max , eff}} \right)^2.$$

This is only valid for an antenna fed at the input point (half wave dipole) $H \leqq \lambda/4$). And

$$R_{s\left(H \leqq \frac{\lambda}{4}\right) = \frac{\pi}{3} \cdot Z_0 \cdot \left(\frac{2H_{\text{eff}}}{\lambda}\right)^2 \; H \leqq \lambda/}$$

Is defined as the radiation resistance of the antenna. For a sinusoidal distribution, not quite correct, we find

$$H_{\text{eff}} = \frac{\lambda}{2\pi \cdot \sin\left(\frac{2\pi}{\lambda} \cdot H\right)} \left[1 - \cos\left(\frac{2\pi}{\lambda} \cdot H\right)\right] \; H < \frac{\lambda}{4}.530/15$$

The radiation resistance only depends now on the effective antenna height H_{eff}, and for $H = \lambda/4$

$R_S\left(\frac{\lambda}{4}\right) = \frac{\pi}{3} \cdot Z_0 \cdot \left(\frac{2 \cdot \frac{2}{\pi} \cdot \lambda/4}{\lambda}\right)^2 = Z_0 \cdot \frac{1}{3 \cdot \pi} \approx 40\Omega$, which really should be 36.6 Ohm

Matching electrically short antennas. X_K circuit. One problem with electrically short antennas is that it is difficult to match the antenna's low base impedance to the feeder cable's real characteristic impedance. The equivalent circuit for an antenna of this type consists of a series connection of the inductance of the vertical part of the antenna L_A, its capacitance C_A, the radiation resistance R_S and the loss resistance R_v. The X_K circuit in Fig. 6.2/5a can be used to match this antenna. The necessary impedance transformation is realized with a reactance two-port network inserted between the cable and antenna. For the equivalent circuit in Fig. 6.2/5b, the matching condition is

$$\frac{1}{Z_L} = j\omega C_K + \frac{1}{j\omega(L_A + L_K) + \frac{1}{j\omega C_A} + R_v + R_s} . \qquad (6.2/14)$$

After separating the real and imaginary parts, equation (6.2/14) leads to

$$\frac{R_v + R_s}{Z_L} + \omega C_K \left[\omega(L_K + L_A) - \frac{1}{\omega C_A}\right] = 1 , \qquad (6.2/15)$$

$$-(R_v + R_s)\omega C_K + \frac{1}{Z_L}\left[\omega(L_K + L_A) - \frac{1}{\omega C_A}\right] = 0 . \qquad (6.2/16)$$

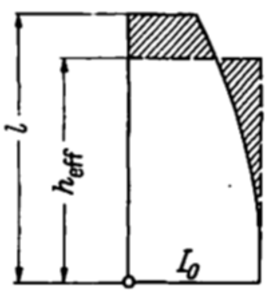
It thus follows that

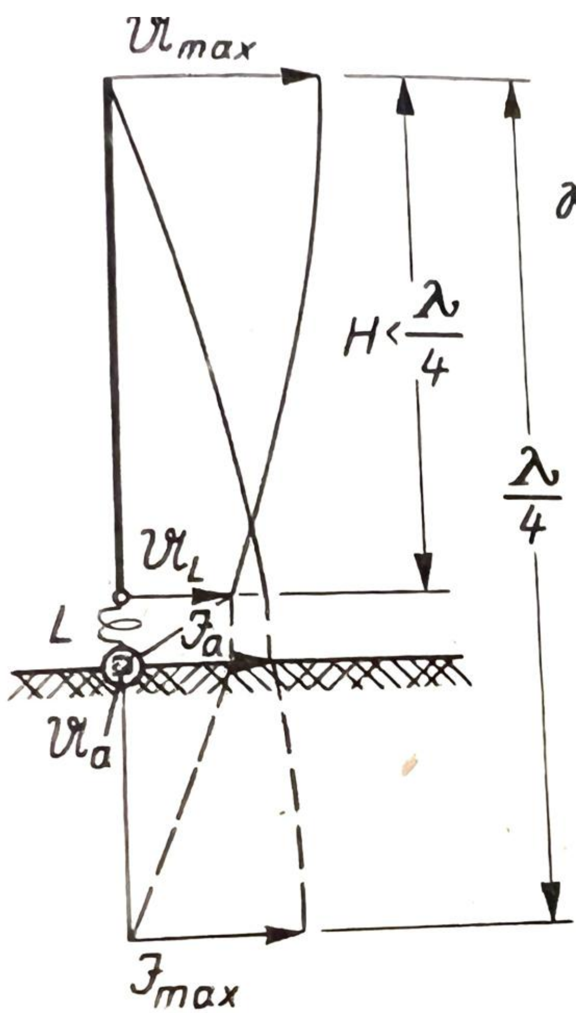
$$\frac{1}{\omega C_K} = \sqrt{\frac{Z_L(R_v + R_s)}{1 - (R_v + R_s)/Z_L}} \approx \left(1 + \frac{R_v + R_s}{2Z_L}\right)\sqrt{Z_L(R_v + R_s)} \qquad (6.2/17)$$

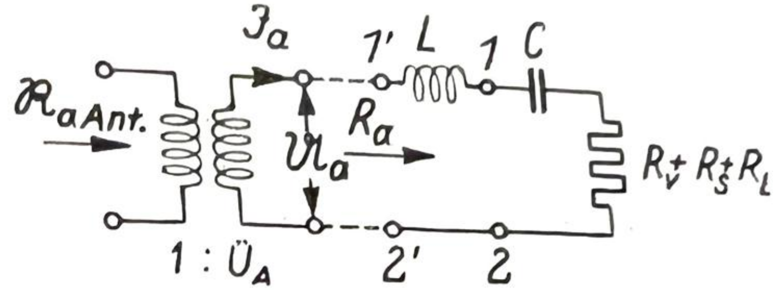
and

$$\omega L_K = \frac{1}{\omega C_A} - \omega L_A + \sqrt{Z_L(R_v + R_s)\left(1 - \frac{R_v + R_s}{Z_L}\right)}$$

$$\approx \frac{1}{\omega C_A} - \omega L_A + \left(1 - \frac{R_v + R_s}{2Z_L}\right)\sqrt{Z_L(R_v + R_s)}\,. \qquad (6.2/18)$$







$$\mathfrak{I}_a = \frac{\mathfrak{U}_a}{R_a}$$

$$\mathfrak{I}_{\max} = \frac{\mathfrak{I}_a}{\sin \frac{2\pi}{\lambda} H}$$

$$|\mathfrak{U}_L| = \Omega L \cdot |\mathfrak{I}_a|$$

$$|\mathfrak{U}_{\max}| = Z \cdot |\mathfrak{I}_{\max}|$$

Fig. 6.2/4. Substitution of an antenna of height *l* with a location-dependent current with an antenna of effective height h_{eff} with constant current

distribution (possible only for $h \leq \lambda/4$)

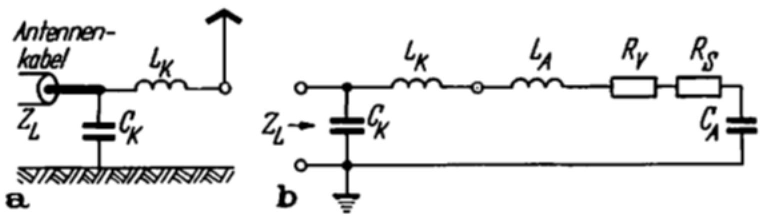
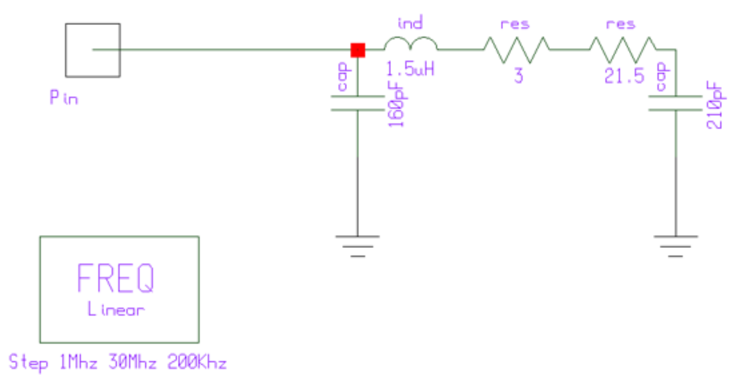
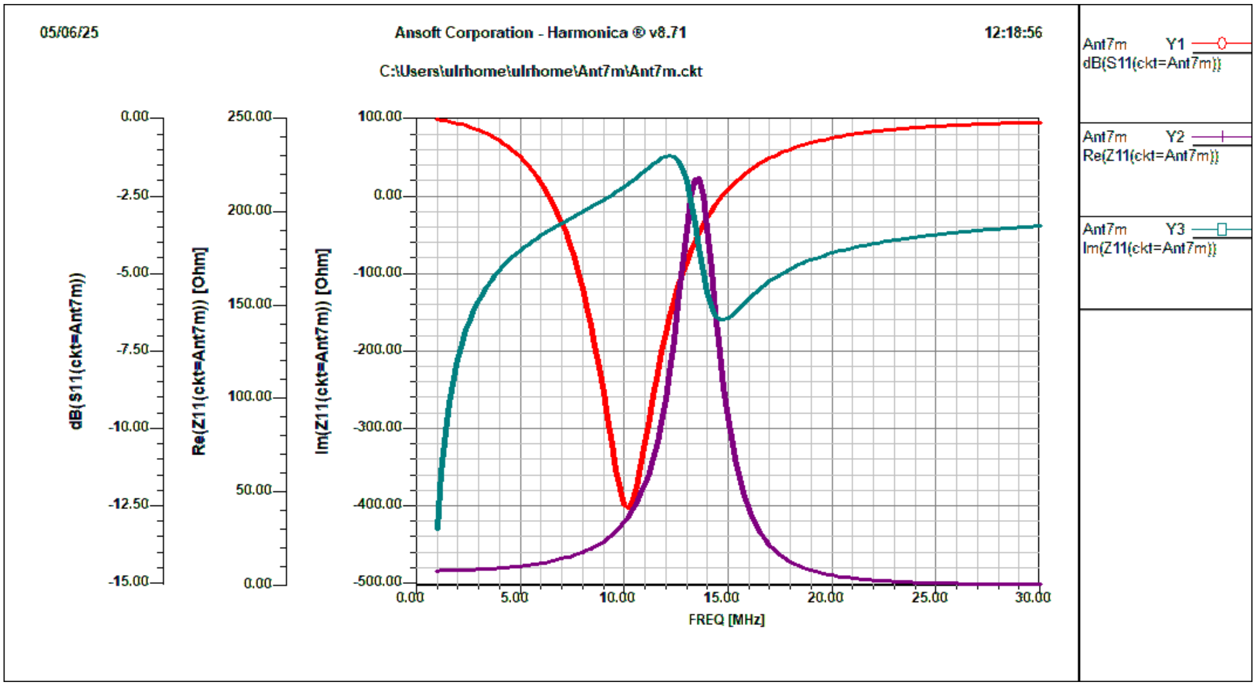


Fig. 6.2/5a,b. Matching the base impedance of a short antenna to the characteristic impedance of a cable. **a** Circuit diagram; **b** Equivalent circuit

For simulation purposes, we use a simplified circuit:





For a 7 m tall antenna with a 5 cm rod diameter, we can assume as follows: $30\ \mathrm{pF/m} = 210\ \mathrm{pF}$, $\mathrm{Rs} = 21\ \Omega$.

This approach using an equivalent circuit for an antenna, $21.5\ \Omega$ radiation resistance and a calculated total value of $30\ \mathrm{pF/m}\ = 210\ \mathrm{pF}$ assumes a linear current distribution, which unless a capacitive hat is used, is wrong.

Interesting is that the resonance according to S11 is slightly above 10 MHz but from the Im (Z11) curve, the resonance is at 9 MHz!

Currents and voltages:

The antenna impedance (not the input impedance) is as follows:

$$Z_{\mathrm{ant}}\ = \ln\left(2\frac{h}{r}\right) Z_0/2\pi = \ln(14/0.05)\cdot\frac{377}{2\pi} = 5.63\cdot\frac{377}{2\pi} \approx 338\ \Omega$$

$R_{\mathrm{load}} = 3\ \Omega$ (losses) + $R_{\mathrm{rad}} = 21.5\ \Omega$ (radiation resistance)

For 100 W transmitter power, we obtain:

$$I_{\mathrm{max}} = \sqrt{100/24.5} \approx\ 2\ \mathrm{A}$$

$$\text{Source voltage} = 2\cdot 24.5 \approx 50\ \mathrm{V}$$

$$\text{Max. antenna voltage} = 338\cdot 2 = 676\ \mathrm{V}$$

The operational Q of the antenna = Zant/R_{load}= 338/21.5~16, 3 dB bandwidth = 625 kHz, and the VSWR will be about 5.8 at this 3 dB power point, 50 % power return.

The efficiency is 21.5/(24.5)= 87.7%.

As we consider operation at various frequencies, the radiation resistance is as follows:

Frequency (MHz)	Radiation resistance (Ω)
5	1
10	4
14.2	8
30.0	36

Electrically short antennas ($l \leq \lambda/8$) over ground (really short)

For the case of short antennas($l \leq \lambda/8$), we can use the approximations $\cos x \approx 1 - x^2/2$ and $\sin x \approx x$ to form a simpler relationship for the electric field strength:

$$E_\vartheta = \mathrm{j}\frac{2\pi \cdot 60\Omega}{r} I_{\mathrm{max}} \frac{\pi l^2}{\lambda^2} \sin\vartheta \mathrm{e}^{-\mathrm{j}\frac{2\pi r}{\lambda}}.$$

If because $l/\lambda \ll 1$ we have

$$I_0 = I_{\mathrm{max}} \sin\frac{2\pi l}{\lambda} \approx I_{\mathrm{max}} \frac{2\pi l}{\lambda},$$

then it follows that

$$E_\vartheta = \mathrm{j}Z_0 \frac{I_0 l \sin\vartheta}{2\lambda} \frac{\sin}{r} \mathrm{e}^{-\mathrm{j}\frac{2\pi r}{\lambda}}.$$

Unlike the case of the Hertzian dipole, here the current drops linearly to 0; the field strength of the short antenna is thus half that of the Hertzian dipole.

We can now calculate the radiation resistance of the electrically short antenna from E_ϑ:

$$P_\mathrm{s} = \int_0^{\pi/2} \frac{E_\vartheta^2}{Z_0} \cdot 2\pi r^2 \sin\vartheta \,\mathrm{d}\vartheta$$

$$= Z_0 \left(\frac{I_0 l}{2\lambda}\right)^2 \cdot 2\pi \int_0^{\pi/2} \sin^3\vartheta \,\mathrm{d}\vartheta$$

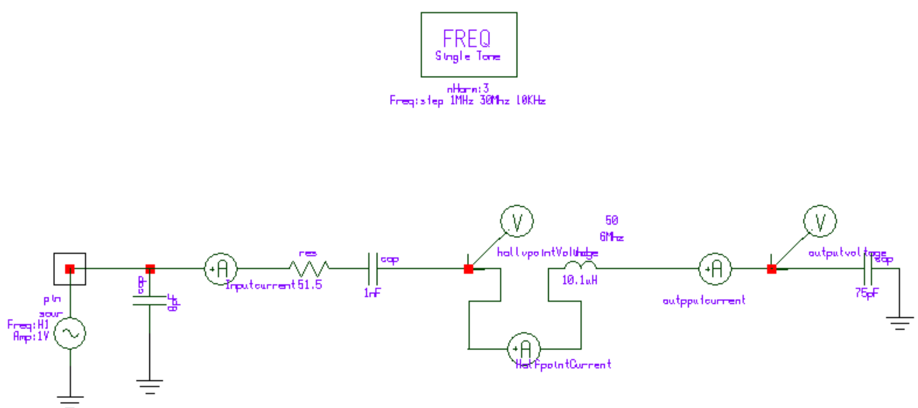
$$R_\mathrm{s} = 60\pi^2\Omega \left(\frac{l}{\lambda}\right)^2 \int_0^{\pi/2} \sin^3\vartheta \,\mathrm{d}\vartheta$$

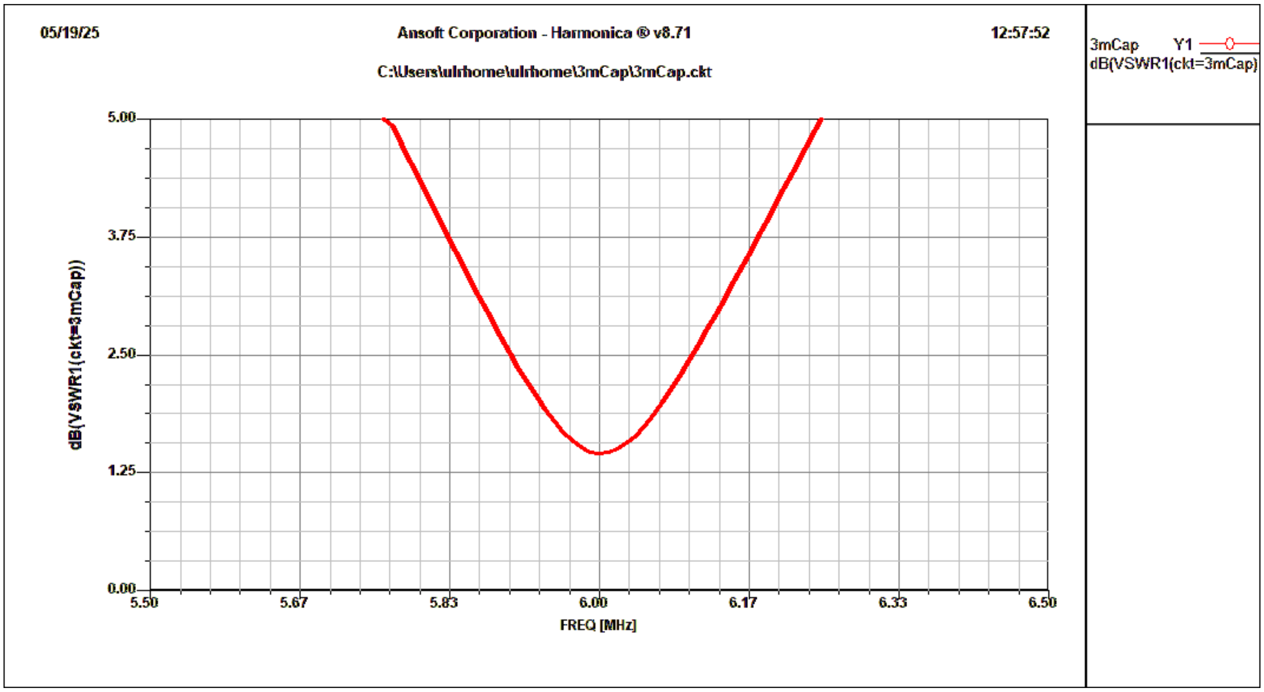
$$R_\mathrm{s} = 40\pi^2\Omega \left(\frac{l}{\lambda}\right)^2 = 395\Omega \left(\frac{l}{\lambda}\right)^2$$

Example: $\lambda = 50\text{m}(f = 6\text{MHz}); l = 3\text{ m}$. Thus, $\frac{l}{\lambda} = 0.06$, $R_S = 1.4\Omega$.

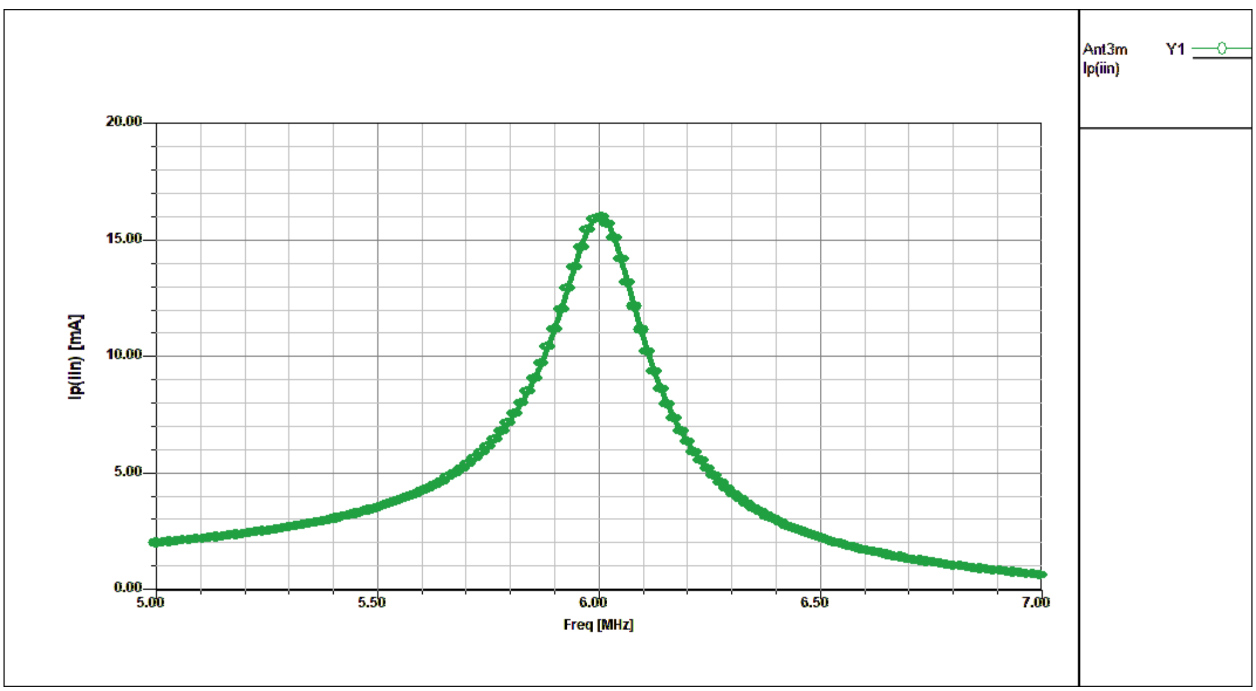
Efficiency = 3/ (3+1.4)=0.68%

Now with loading coil :

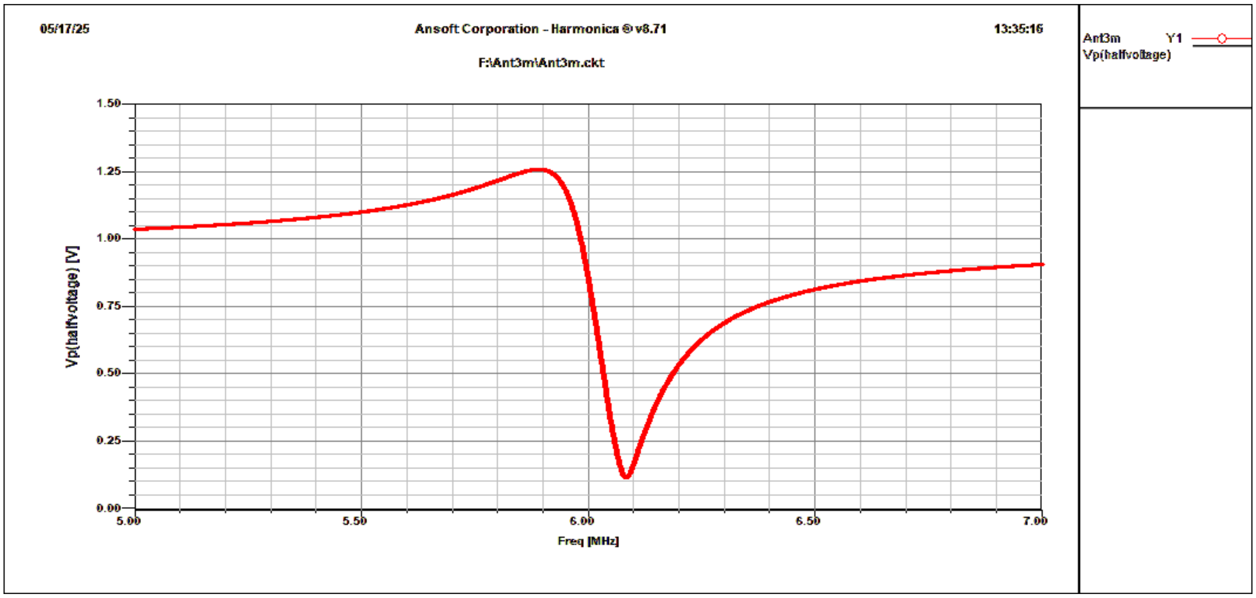




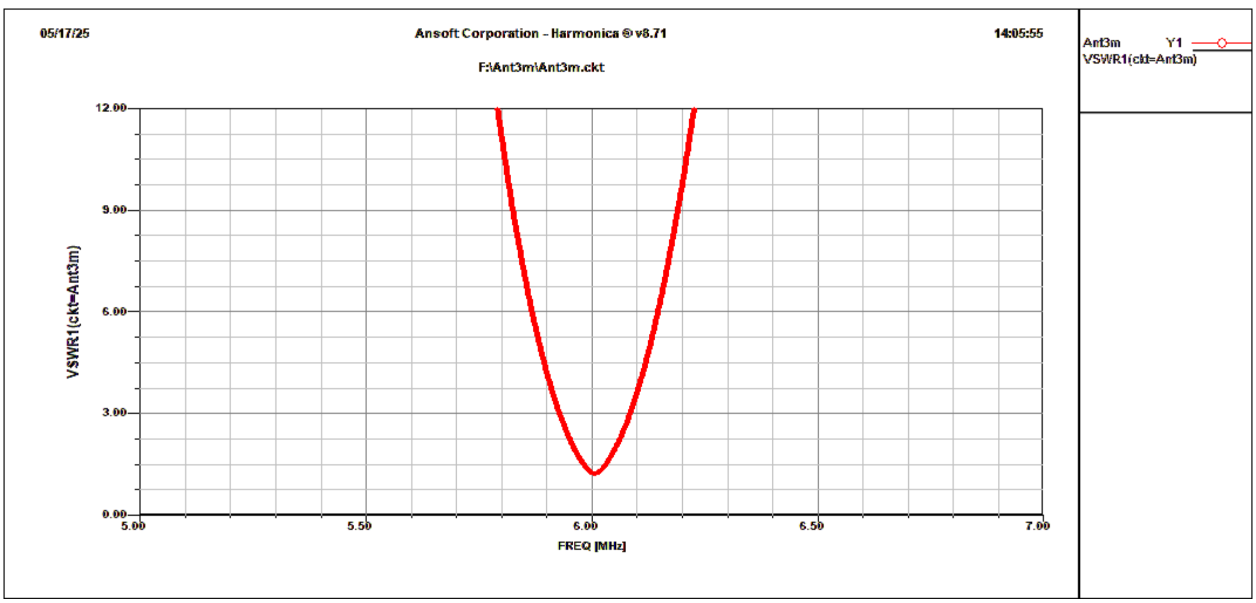
Above is the circuit for the antenna matching



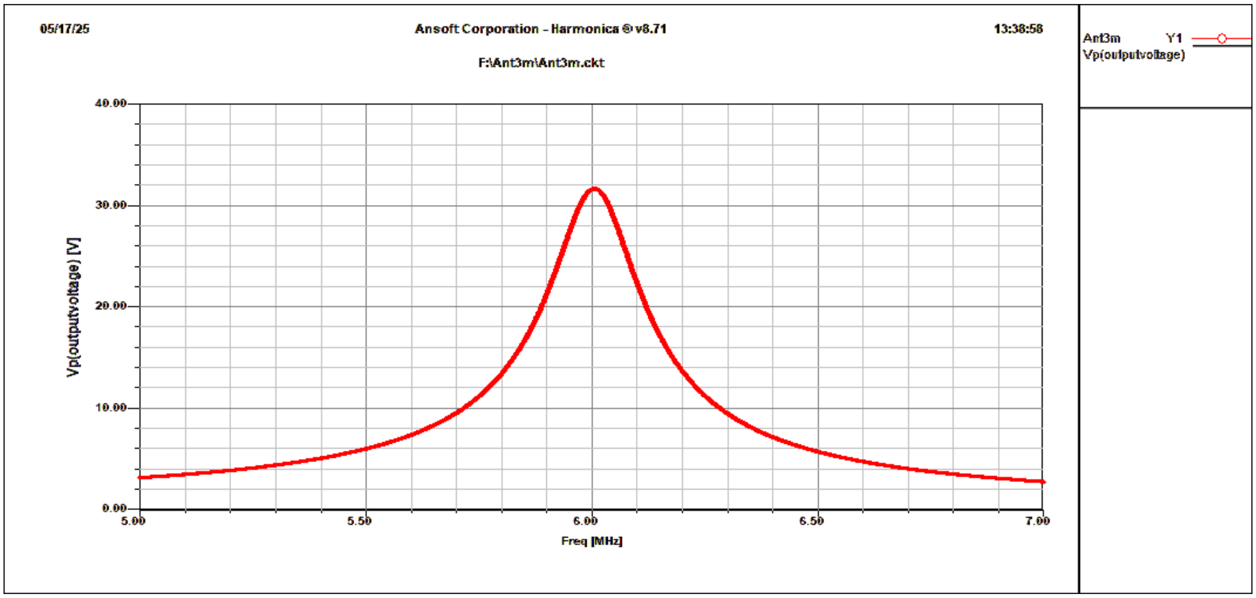
Zin 1 V : 16mA =62.5 Ohm.



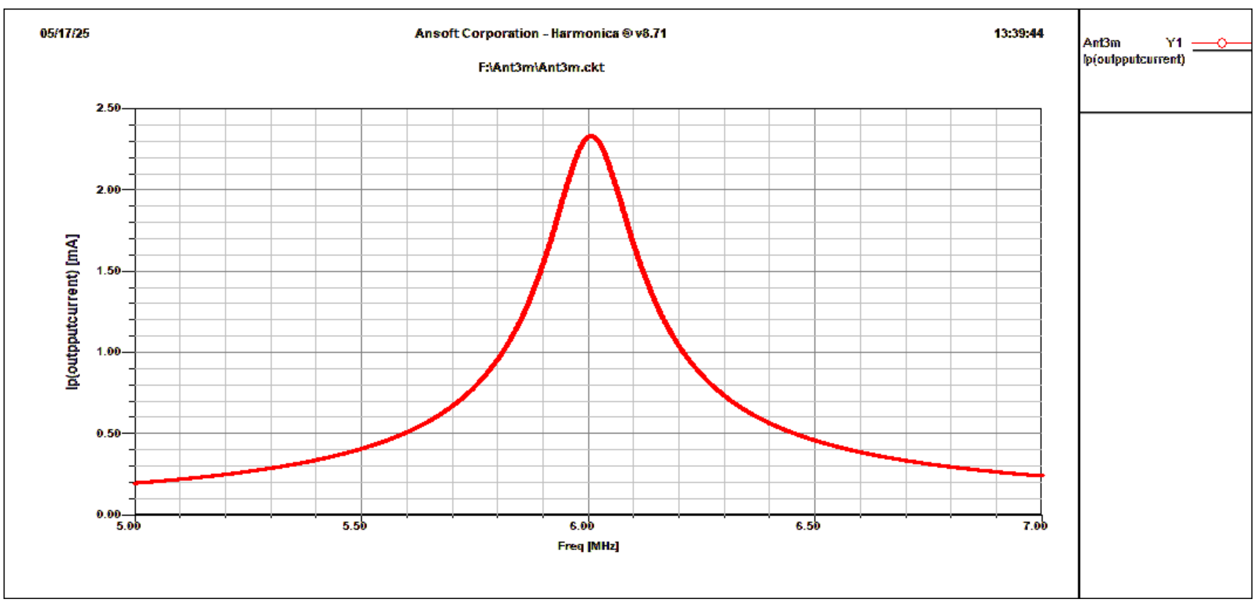
The phase turns after 2 m, first transmission line



Predicted VSWR of the "matched" antenna"



Output voltage probe

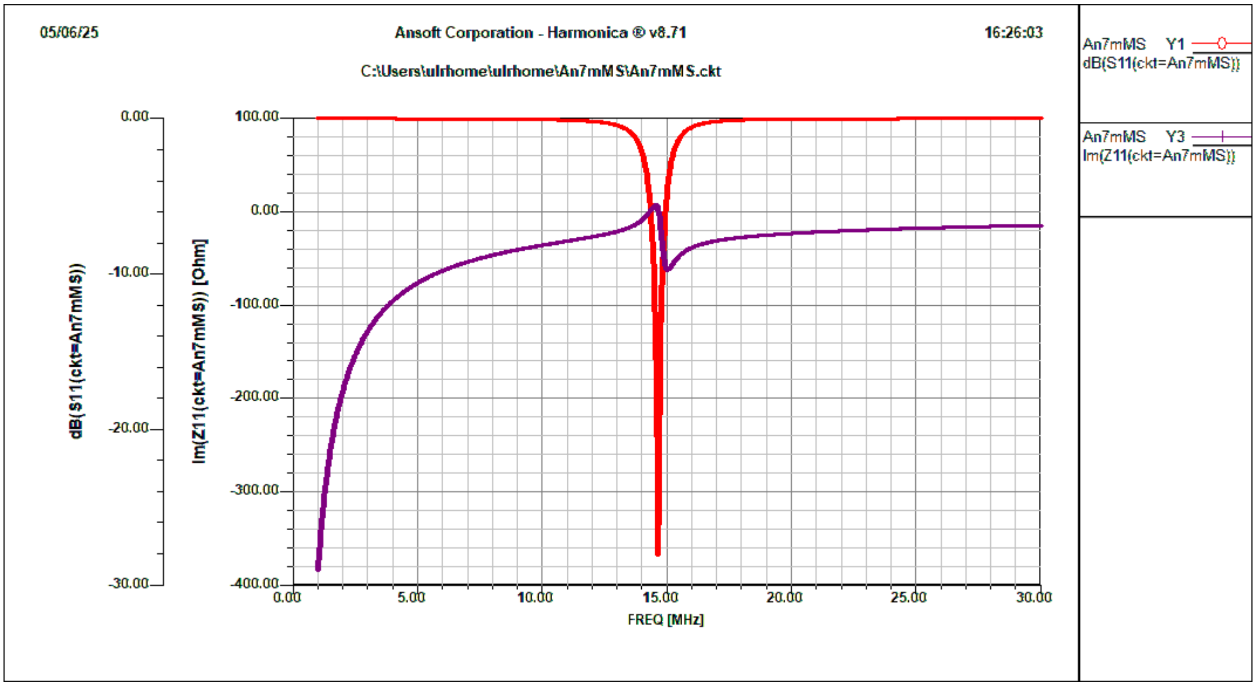


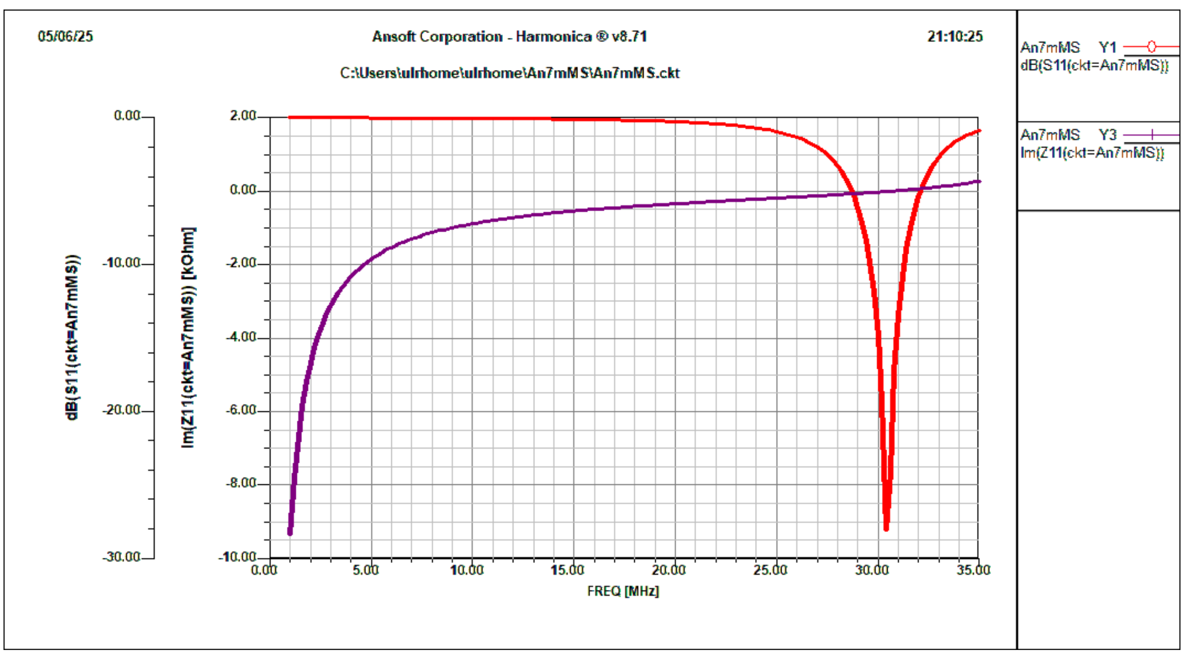
Output current probe

Antenna end impedance = 31.5V/2.33mA=13.5 kOhm.

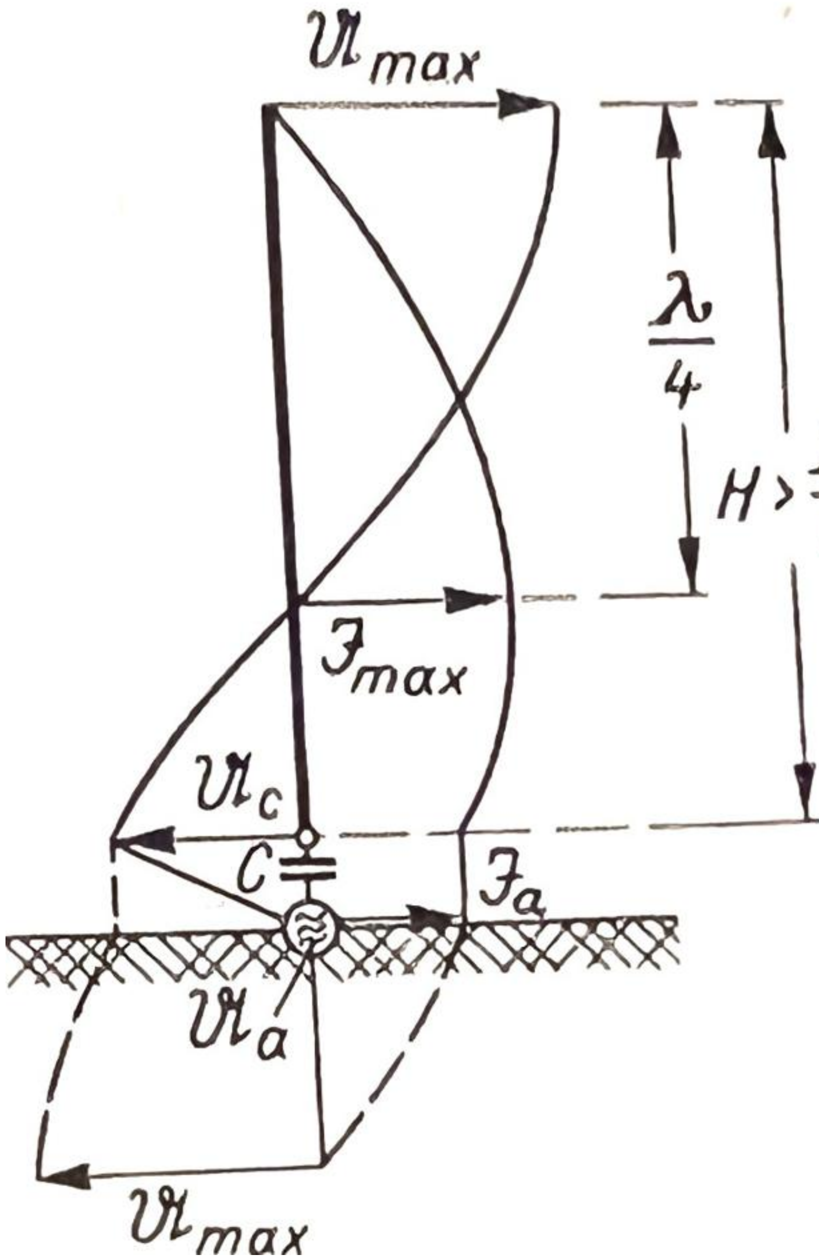
3dB bandwidth = half power = (6.17-5.86) = 300 kHz

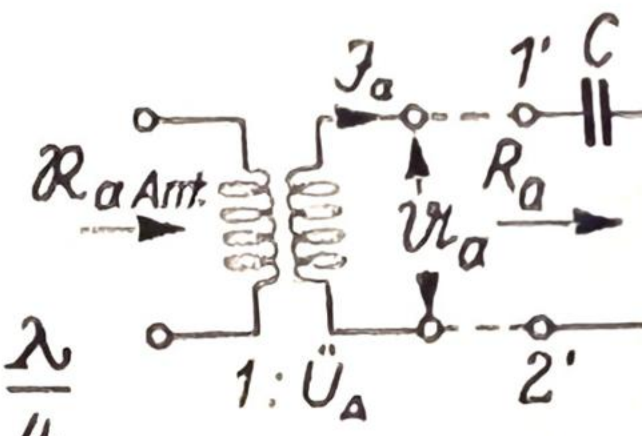
Other frequencies:





This requires a different matching network as the operating frequency is now above self resonance.



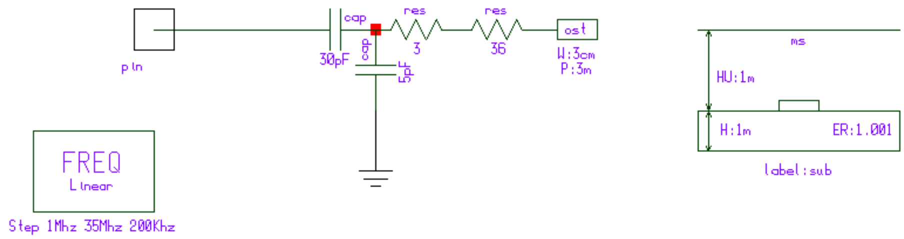


$$\mathfrak{J}_a = \frac{\mathfrak{U}_a}{R_a};$$

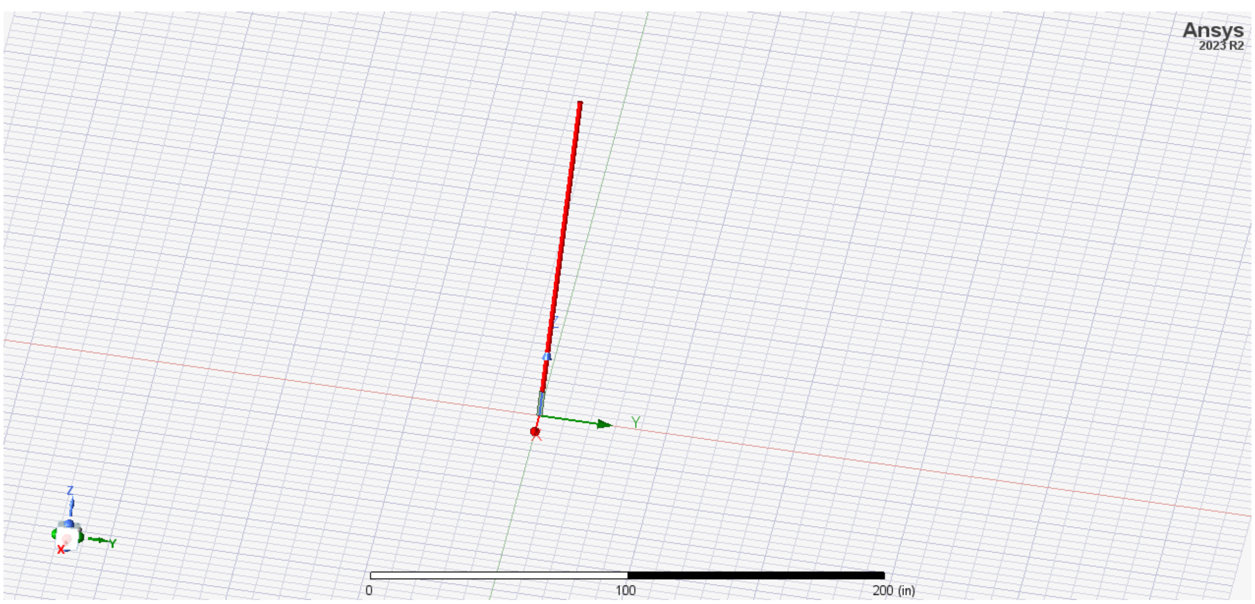
$$\mathfrak{J}_{max} = \frac{\quad}{\sin}$$

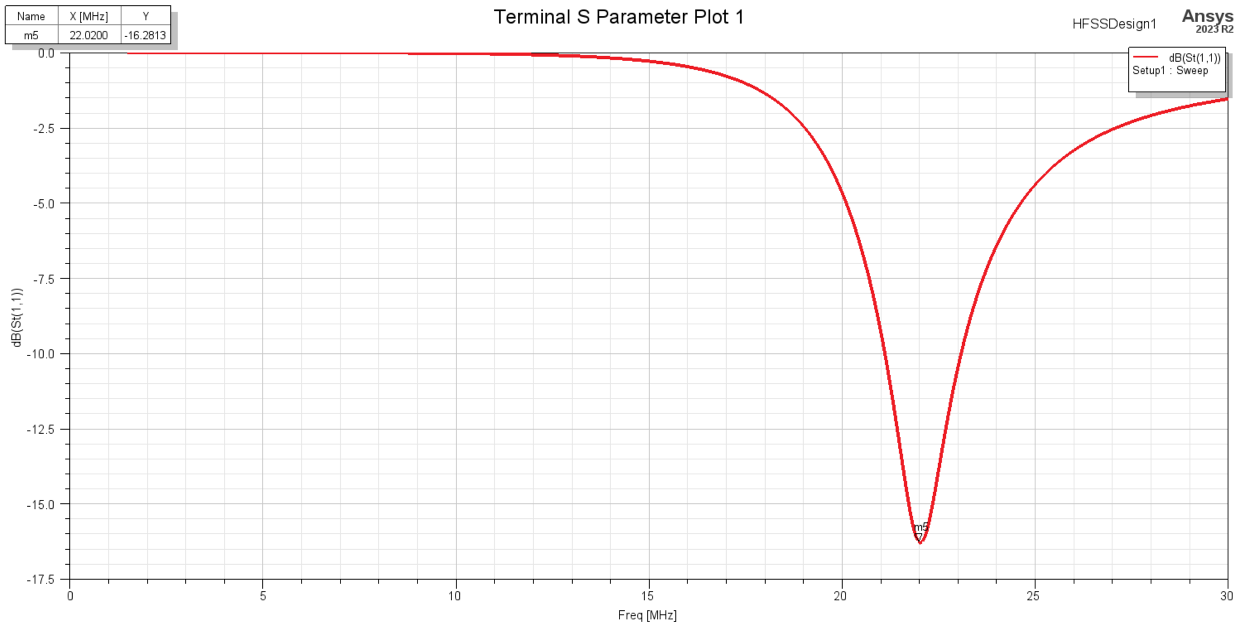
$$|\mathfrak{U}_c| = |\mathfrak{J}_a|$$

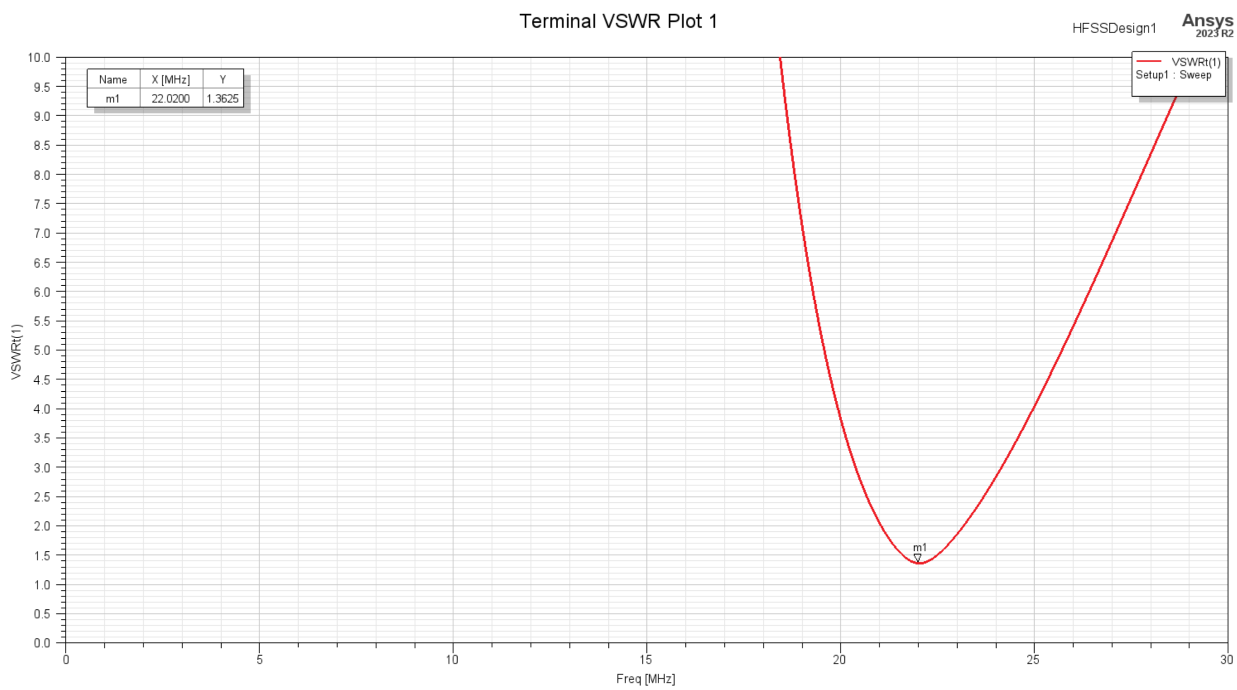
$$|\mathfrak{U}_{max}| = Z \cdot$$

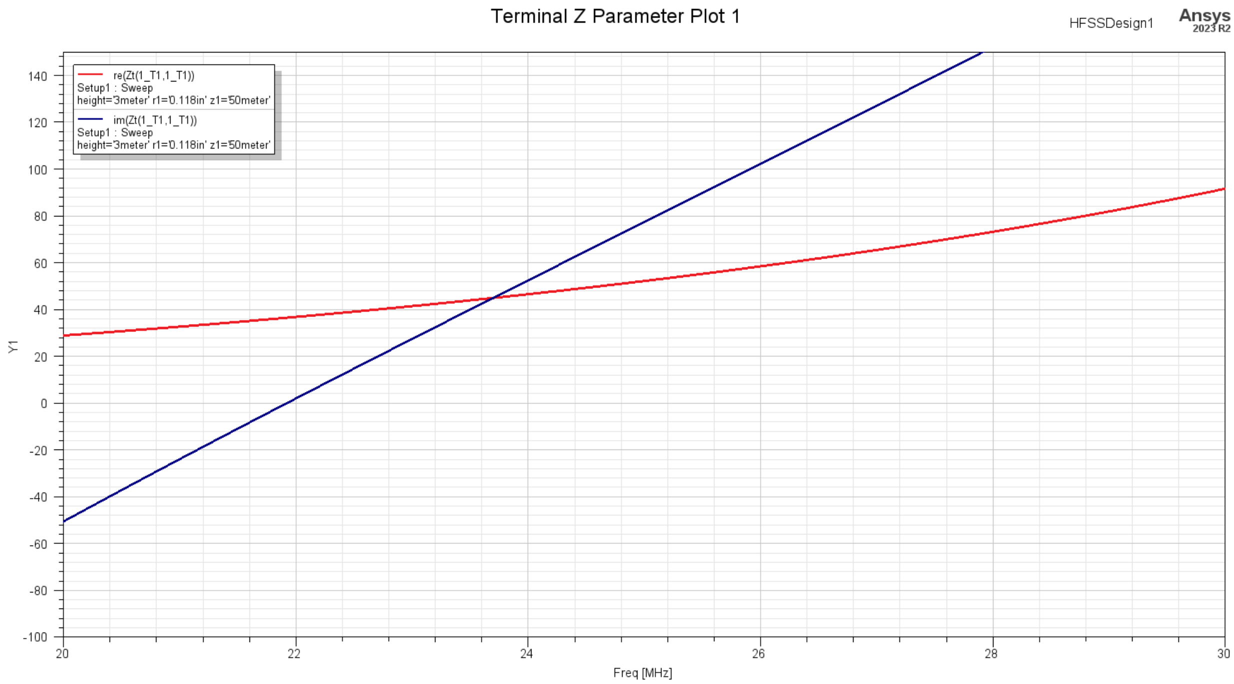


Now the 3 D simulation using HFSS:




Terminal S Parameter Plot 1
HFSSDesign1
Ansys
2023 R2
Name
X [MHz]
Y
m5
22.0200
-16.2813
dB(St(1,1))
Setup1 : Sweep
dB(St(1,1))
Freq [MHz]
m5
0.0
-2.5
-5.0
-7.5
-10.0
-12.5
-15.0
-17.5
0
5
10
15
20
25
30


Terminal VSWR Plot 1
HFSSDesign1
Ansys
2023 R2
Name
X [MHz]
Y
m1
22.0200
1.3625
VSWRt(1)
Setup1 : Sweep
VSWRt(1)
Freq [MHz]
m1
10.0
9.5
9.0
8.5
8.0
7.5
7.0
6.5
6.0
5.5
5.0
4.5
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
0
5
10
15
20
25
30



At resonance (Imaginary part=0), the radiation resistance is predicted to be close to 36 Ohm!

Appendix: A radiation resistance calculation from the past (not quite correct)

Radiation resistance of an antenna

To calculate the radiation resistance, we determine the radiated power through a very large sphere and then divide this value by I^2. We use the Poynting vector to determine the radiation:

$$S = [E\ H].$$

1. Dipole in free space. We calculated the electric and magnetic field strengths as follows:

$$E = \frac{\mu_0 c}{2}\frac{Ih}{r\lambda}\sin\vartheta; \quad H = \frac{Ih}{2r\lambda}\sin\vartheta.$$

The power is then

$$P = \frac{1}{T}\int_0^T \int_0^{180^\circ} 2\pi r^2 \sin\vartheta d\vartheta\ [E \cdot H]\ dt.$$

We will assume the antenna is so short that the waves arriving from different parts of the antenna – even at different angles – do not have a noticeably different delay. Thus, we can simply add up the fields without taking the phase shift into account. Instead of

$$H = \frac{I}{2\lambda}\int \frac{e^{j\omega\left(t-\frac{r}{c}\right)}}{r} ds,$$

we can write:

$$\frac{Ie^{j\omega\left(t-\frac{r}{c}\right)}}{2\lambda r}\int_0^h ds = \frac{Ih}{2\lambda r}e^{\cdots}.$$

The power is then:

$$P = \frac{I^2 h^2}{r^2\lambda^2}\frac{\mu_0 c}{4} 2\pi r^2 \int_0^{180^\circ} \sin^3\vartheta d\vartheta. \quad P = \text{ Average power}; \; I = \text{ RMS current.}$$

If we calculate the integral

$$\int_0^{180^\circ} \sin^3\vartheta d\vartheta = -\int_0^{180^\circ} (1-\cos^2\vartheta) d\cos\vartheta = 2 - \frac{2}{3} = \frac{4}{3},$$

we obtain:

$$P = \frac{2\pi\mu_0 c}{4}\frac{4}{3}\frac{h^2 I^2}{\lambda^2} = \frac{80\pi^2 h^2 I^2}{\lambda^2}\ \mathrm{W},$$
$$\left(\frac{2\pi\mu_0 c}{3} = \frac{2\pi}{3}4\pi 10^{-7}\frac{\mathrm{V\,s}}{\mathrm{A\,m}}3\cdot 10^8\frac{\mathrm{m}}{\mathrm{s}} = 80\pi^2\ \mathrm{Ohm}\right).$$

2. If the antenna is positioned on the ground, then, as previously noted, we need to substitute $2h$ for h and, since the radiation is limited to one hemisphere, we divide the result by 2. We obtain:

$$P = 160\pi^2\frac{h^2 I^2}{\lambda^2}; \quad R_{\text{rad}} = \frac{P}{I^2} = 160\pi^2\frac{h^2}{\lambda^2}\mathrm{Ohm} = 1580\frac{h^2}{\lambda^2}\mathrm{Ohm}.$$

3. If the antenna is not top-loaded and has a length of $\lambda/4$, the current is distributed sinusoidally:

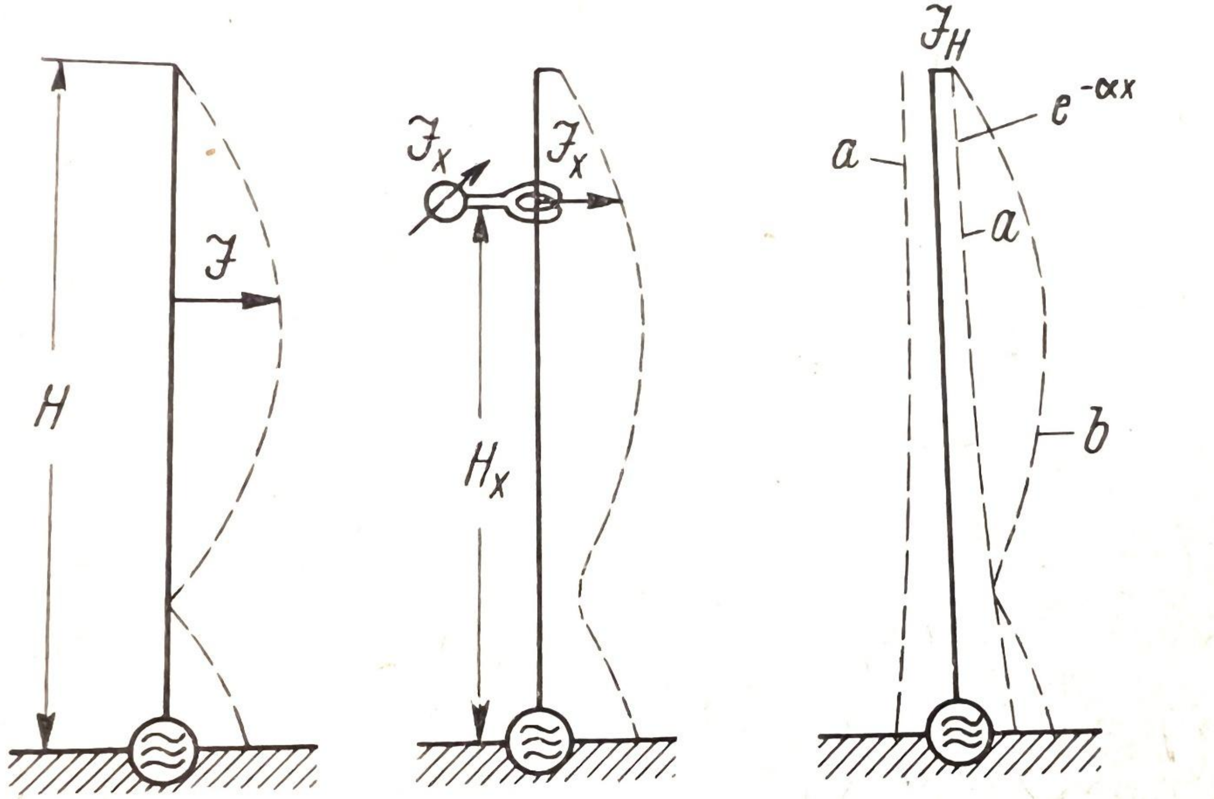
$$I = I_0\sin\frac{2\pi s}{\lambda}, \qquad \int_0^{\lambda/4} I ds = \frac{I_0\lambda}{2\pi}, \quad h_{\text{eff}} *= \frac{\lambda}{2\pi}.$$

Substituting the value $1/2\pi$ for h_{eff}/λ, we obtain $R_{\lambda/4} = 40\ \mathrm{Ohm}$.

Our previous assumption of phase equality is no longer applicable here because the waves arriving vertically upwards from the base point and from the tip of the antenna have a 90° phase shift. However, this phase shift has an effect only in the low-radiation locations in the zenith. Strict calculation of the integral thus leads to a somewhat smaller value of 36.6 Ω.(At the time the calculation of the effective height was wrong). The antenna end current was not zero, and the integral:

$$\int_{0}^{180^\circ} \sin^3 \vartheta d\vartheta$$

was not properly solved with the necessary fine resolution



The small ohmic resistance of the antenna wire and the more significant grounding resistance also enter into the radiation resistance.

Reference:

ELECTROMAGNETIC WAVES AND THEIR APPLICATIONS AT MICROWAVE FREQUENCIES

Raghunath K. Shevgaonkar and Ulrich L. Rohde

Springer 2026