Oriented random rotations for the modelling of fiber-reinforced materials

STUDENT PROJECT

Willi Helbig, Ivo Steinbrecher[1] and Alexander Popp[1]
[1]Institute for Mathematics and Computer-Based Simulation, University of the Bundeswehr Munich, Germany

Universität der Bundeswehr München

Outline

Motivation

Randomly placing fibers in concrete has proven to be beneficial for increasing stability and stress resistance. Simulation of such structures is of vital importance during the development stage. The challenge is to generate uniform rotations and control the parameters and distributions around a preferred direction, to mimic real-life distribution of fibers.

Displaying distribution of rotations and vectors

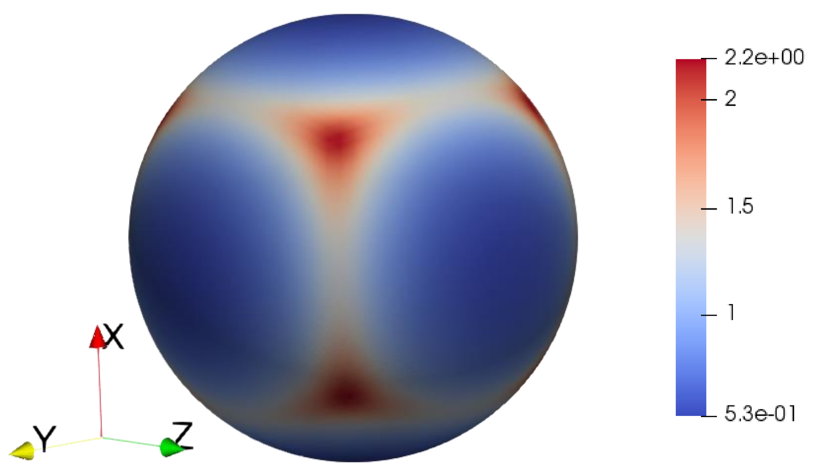
To display random vectors, we generate 10^8 points on a unit sphere and compare the calculated density to a perfect uniform distribution.

To display random rotations, we take a given vector (e.g. $\boldsymbol{e}_1$) and apply the random rotations to this vector. After that, we plot the distribution of this rotated vector in comparison to a perfect uniform distribution.

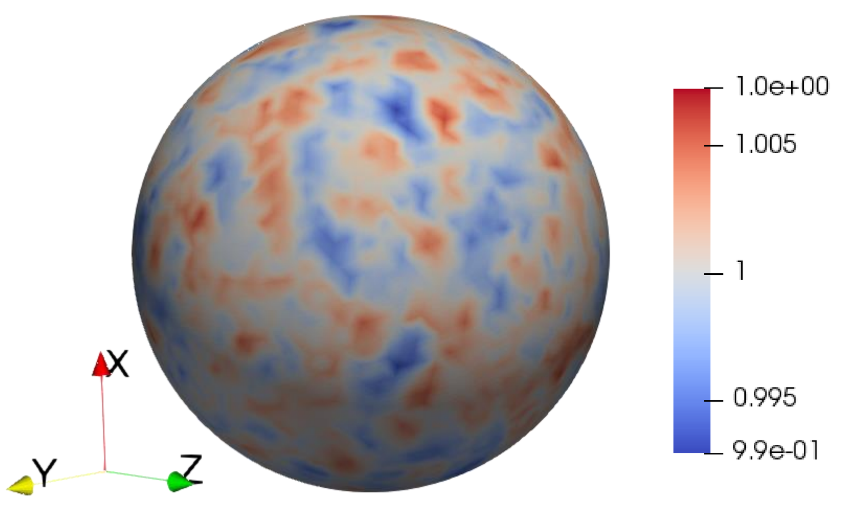
Uniform vectors and rotations

Random vectors on a unit sphere

Generating random vectors in a volume, with a given interval $[-1,1]^3$ proves to be unsuited for producing a random outcome on the unit sphere. Distribution density increases, where the corners of the modeled cube would be.



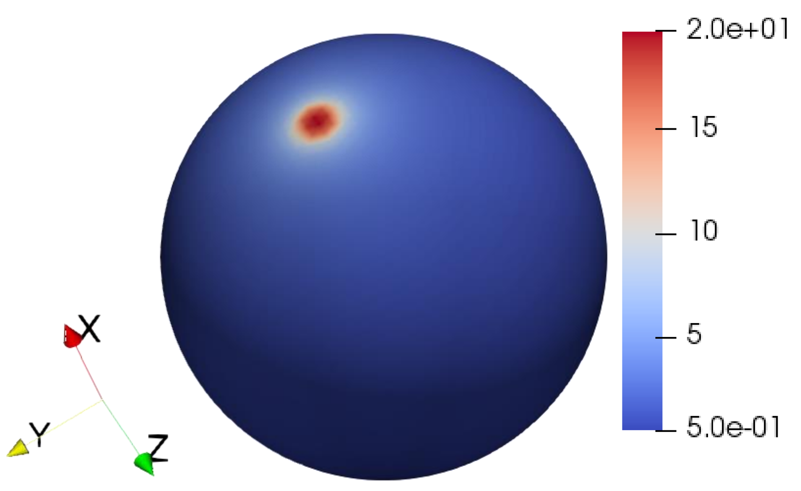
To overcome this, all vectors with a length greater than one are discarded. This results in a uniform distribution.



Uniform random rotations

Rotations from random axes and angle

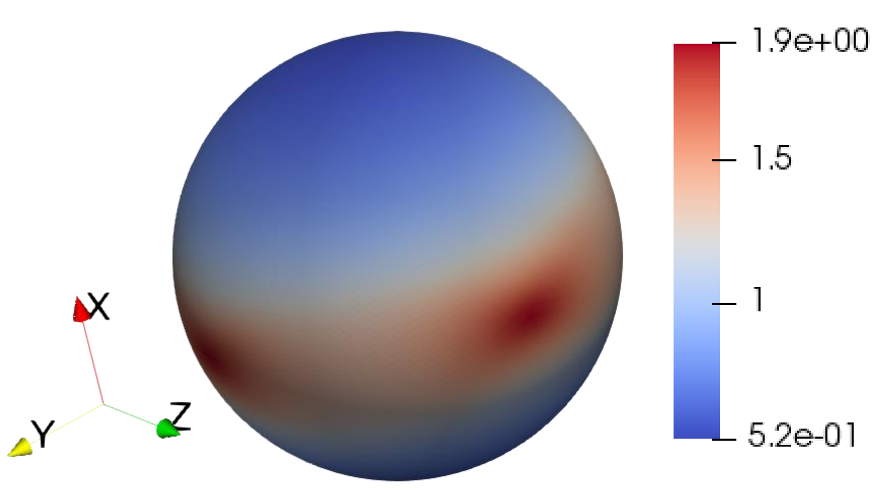
One could try to choose a random axes and angle to generate a random rotation, but this results in a non-uniform distribution.



Distribution density of $\boldsymbol{e}_1$

Rotations from random quaternion

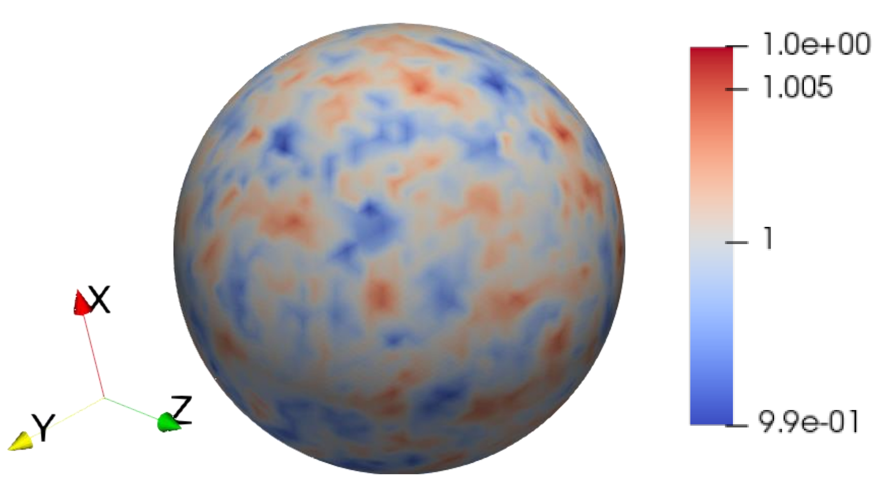
Rotations in 3D can be represented by a quaternion. One could try to choose completely random quaternions, also resulting in non-uniform distribution.



Distribution density of $\boldsymbol{e}_1$

Rotations from quaternions on 4D sphere

Taking the same approach as in $\mathbb{R}^3$ before, quaternions with a norm greater than 1 are discarded. This eliminates the quaternions which would be outside the 4D-sphere and gives the desired outcome of a uniform distribution.

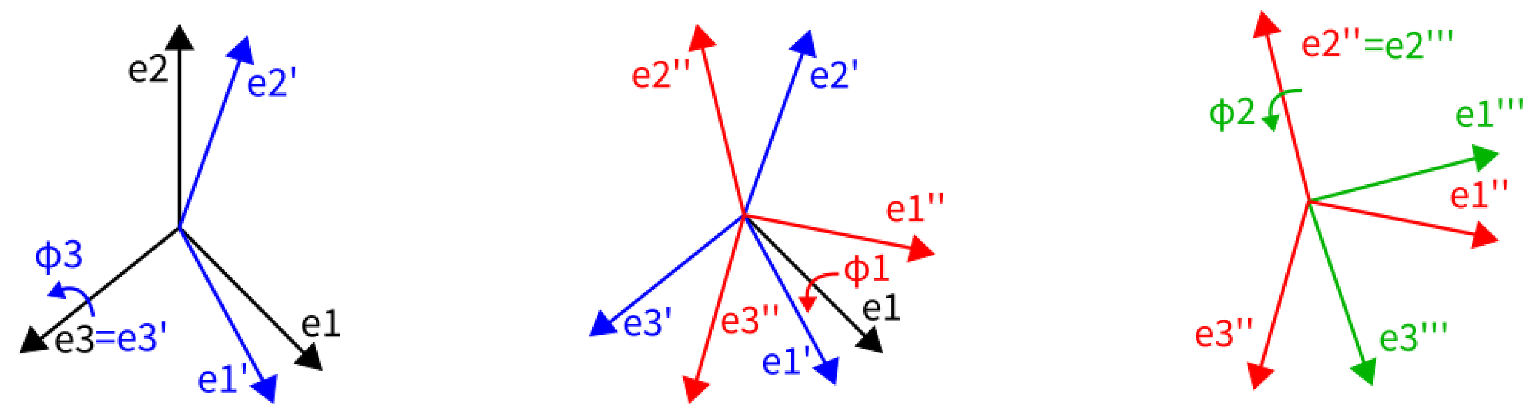


Distribution density of $\boldsymbol{e}_1$

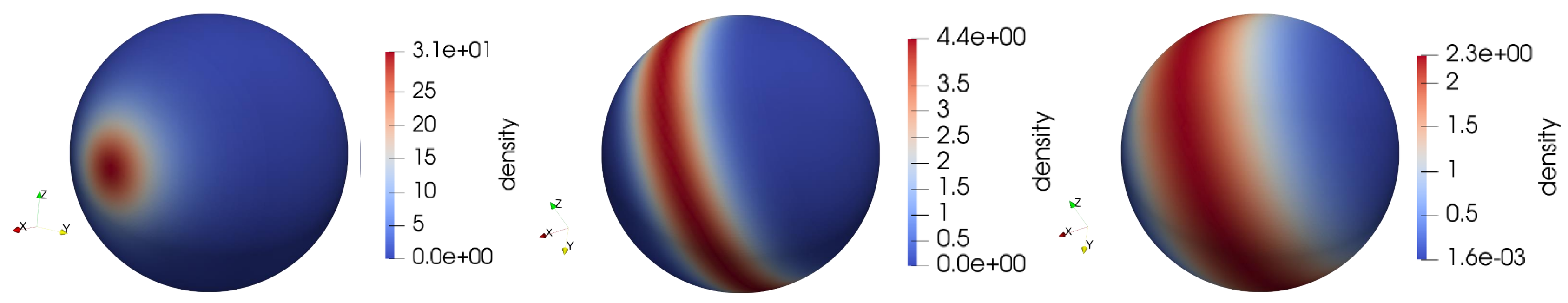
Random rotations with preferred directions

Rotating the coordinate system

Generating vectors with a preferred direction is done through three successive rotations, with $\boldsymbol{e}_1$ representing said direction.
Firstly, $\boldsymbol{e}_3$ is the rotation axis and φ_3 (normal distribution, $\sigma_3 = 10°, \mu_3 = 0°$) the rotation angle.
Secondly, $\boldsymbol{e}_1$ is the rotation axis and φ_1 (uniform distribution, [-π,π]) the rotation angle.
Lastly, $\boldsymbol{e}_2''$ is the rotation axis and φ_2 (normal distribution, $\sigma_2 = 20°, \mu_2 = 0°$)



Resulting coordinate systems after each rotation



Final distributions for $\boldsymbol{e}_1$, $\boldsymbol{e}_2$, and $\boldsymbol{e}_3$

Application

Implementing results

With the previously described generation of oriented random rotations a fiber-reinforced concrete block is modeled and simulated, shown in the left picture. The predominant direction is the x-axis, with the rotation angles as being described above. The block is loaded with a pressure load in the x-direction and the resulting interface traction is shown in the right picture.