



INSTITUTE OF
SPACE TECHNOLOGY & **SPACE APPLICATIONS**

der Bundeswehr
Universität München

Session B5: Receiver Design, Signal Processing, and Antenna Technology 1

Receiver Clock Estimation for RTK-Grade Multi-GNSS Multi-Frequency Synthetic Aperture Processing (SAP)

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Motivation

- Why synthetic aperture processing (SAP), why ultra-low bandwidth PLL?

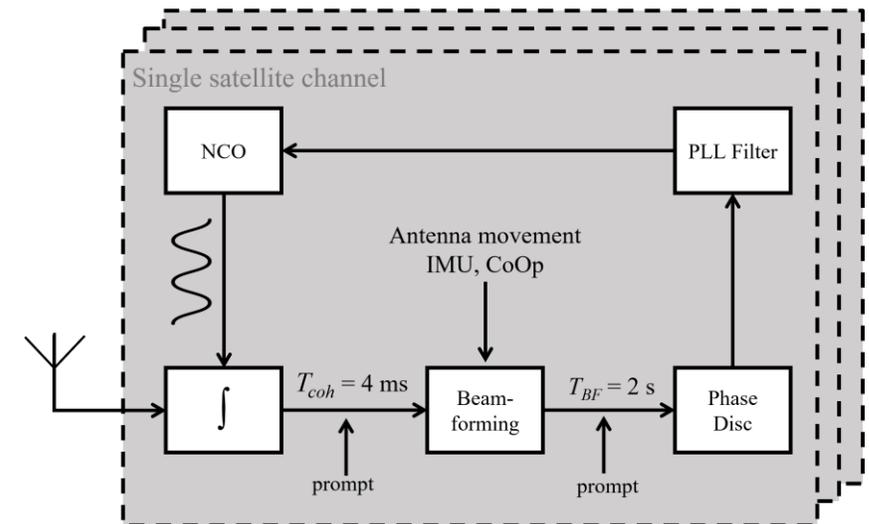
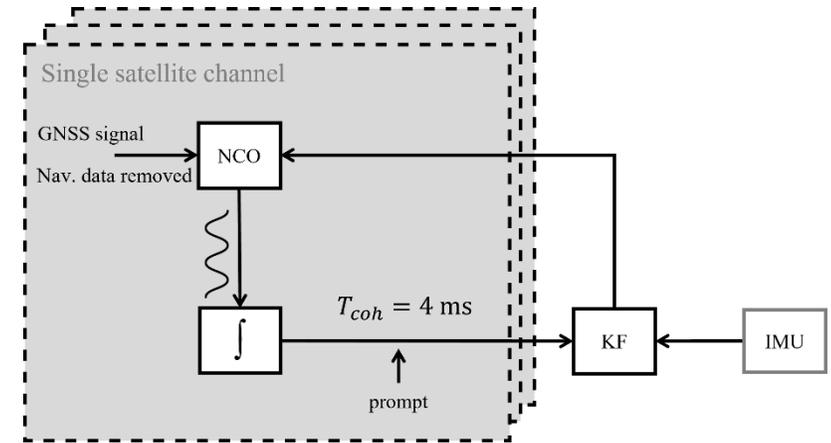
- increased tracking robustness
- increased sensitivity
- increased position accuracy

- Goal: ➤ RTK-grade GNSS observations!
- Overcome signal degradation, e.g. canopies, with fixed carrier-phase ambiguities

- Other works on SAP, ultra-Low-bandwidth PLL:
 - Co-Op tracking for carrier phase (Zhodzishsky M. et al., 1998)
 - GNSS Synthetic Aperture Processing with Artificial Antenna Motion (Pany T. et al., 2013)
 - “*Supercorrelation*” from FocalPoint
 - Improving GNSS carrier phase tracking using a long coherent integration architecture (Feng X. et al, 2022)
 - ...

Alternative Approaches: SAP or Deep Coupling

- Deep Coupling
 - Kalman Filter (KF) receives prompt correlator and IMU data
 - State vector includes clock states
 - KF parameters determine effective bandwidth as main parameters
- Synthetic Aperture Processing (SAP)
 - Prompt correlator values are captured at different spatial locations due to antenna movement
 - Beam-forming algorithm combines prompt correlator values making us of prior knowledge of antenna movement (IMU-based) and clock jitter (from CoOp loop)
 - Beam-forming results in longer coherent integration times for beam-formed prompt correlator, driving conventional PLL
 - Beam-forming interval time, strategy and PLL bandwidth main parameters



Thermal Noise Analysis (Aided PLL vs. SAP)

- Definition of phase discriminator

- $e_\phi = \text{angle}\{P\}$
 - e_ϕ ... 4-quadrant phase discriminator
 - P ... complex valued prompt correlator based on an integration time of T_{coh}
- Note: independent of amplitude, but affected by squaring loss
- Under AWGN, e_ϕ is of zero mean with a variance of $\sigma_{e_\phi}^2 = \frac{1}{2C/N_0 T_{coh}} \left(1 + \frac{1}{2C/N_0 T_{coh}}\right)$

- Aided PLL

- Carrier phase noise is determined by PLL loop bandwidth: $\sigma_{PLL}^2 = 2B_{PLL} T_{coh} \sigma_{e_\phi}^2 = \frac{B_{PLL}}{C/N_0} \left(1 + \frac{1}{2C/N_0 T_{coh}}\right)$

- Synthetic Aperture Processing (SAP)

- Beam-forming strategy: maximize line-of-sight signal power
- Assumption: antenna movement and clock jitter perfectly known
- Thus: $P_{BF} = \int_{t=t_0}^{t_0+T_{BF}} P(t) dt$; P_{BF} ... beam-formed prompt correlator, T_{BF} ... beam-forming interval
- $e_{\phi,BF} = \text{angle}\{P_{BF}\}$; $e_{\phi,BF}$... beam-formed phase discriminator
- Including PLL, we obtain in a carrier phase noise of $\sigma_{PLL,BF}^2 = \frac{B_{PLL}}{C/N_0} \left(1 + \frac{1}{2C/N_0 T_{BF}}\right)$

- Conclusion:

- Aided PLL and SAP show almost the same thermal noise performance; SAP slightly better due to decreased squaring loss

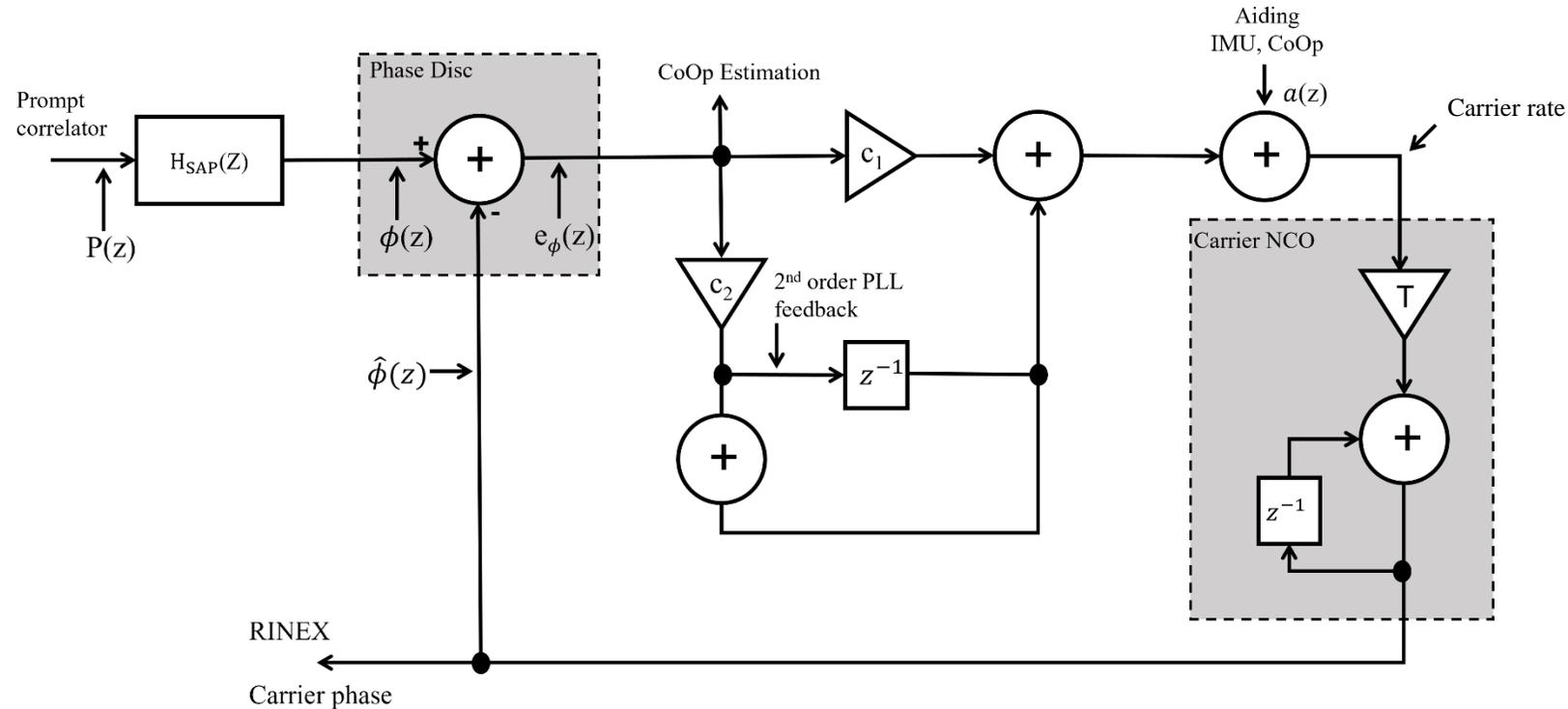
Multipath Analysis of Aided PLL

- Phase discriminator model under influence of clock oscillation and multipath (neglect noise, assume perfect code phase and Doppler lock)

$$e_{\phi}(t) = \text{angle}\{P(t)\} = \text{angle}\left\{ e^{\frac{2\pi j}{\lambda}(\phi(t) + a_c \sin \omega_c t - \hat{\phi}(t))} + \beta e^{\frac{2\pi j}{\lambda}(\phi(t) + a_c \sin \omega_c t + v_{mp} t - \hat{\phi}(t))} \right\}$$

- λ ... carrier wavelength [m]
 - $\phi(t)$... true carrier phase [m], including geometry, tropo, iono, etc., but excluding clock
 - a_c ... amplitude of clock oscillation [m]
 - ω_c ... frequency of clock oscillation [Hz]
 - $\hat{\phi}(t)$... estimated carrier phase [m]
 - β ... relative multipath amplitude
 - v_{mp} ... multipath velocity relative to line of sight
- Linearization for phase lock conditions and $\beta \ll 1$
 - $e_{\phi}(t) \approx \frac{1}{\lambda} \left(\phi(t) + a_c \sin \omega_c t - \hat{\phi}(t) \right) + \beta \sin \left(\frac{v_{mp} t}{\lambda} \right)$
- **Conclusion**
 - For small multipath amplitudes and good phase tracking conditions, PLL tracking can be analyzed in frequency domain

z-Transform of Channel Structure (aided PLL, with and without SAP)



- $\phi(z)$... true carrier phase
- $\hat{\phi}(z)$... estimated carrier phase
- $e_{\phi}(z)$... phase discriminator
- $a(z)$... rate aiding, based on IMU, CoOp
- $H_{SAP}(z)$... beamforming ($H_{SAP}(z)=1$ for standard PLL)
- T_{coh} ... coherent integrate time (e.g. 4 ms)
- c_1 ... 2nd order PLL coefficient ($= 1.89\sqrt{2}B_{PLL}$)
- c_2 ... 2nd order PLL coefficient ($= (1.89B_{PLL})^2 T_{coh}$)

Use of PLL-Discriminator for Clock-Estimation

- True carrier phase includes
 - Geometry, iono, tropo,
 - receiver/satellite clock
- Task of CoOp is to estimate clock jitter
 - Common to all signals
 - High frequency
 - Does not include clock drift or other low frequency processes
- Transfer function from true carrier phase to phase discriminator

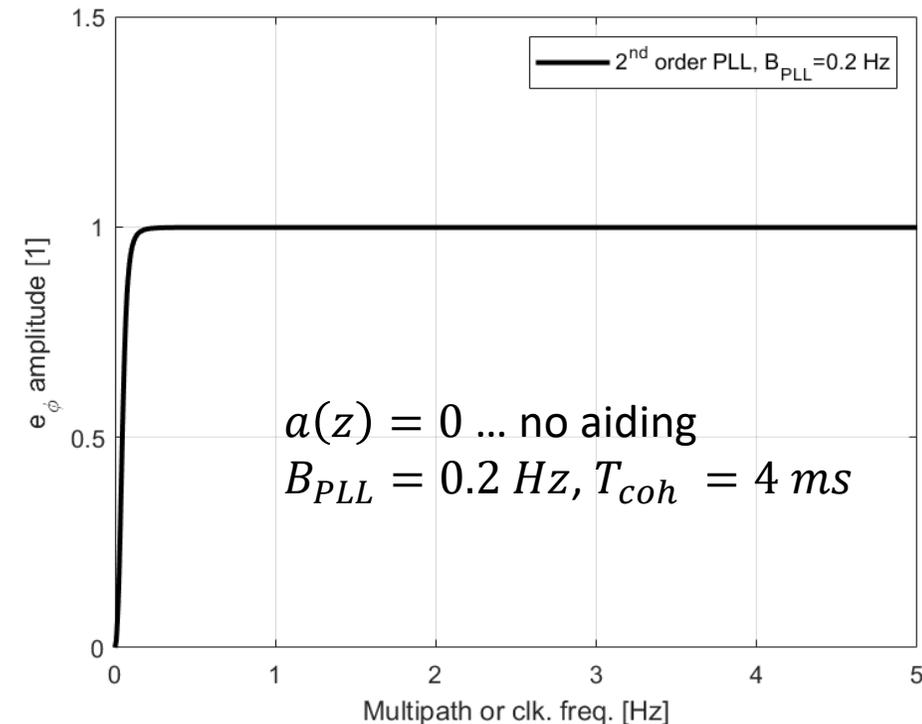
- $e_\phi(z) = \phi(z)D(z) + A(z)a(z)$

- $D(z) = \frac{-1+2z-z^2}{1+z(T_{coh}(c_2-c_1)-2)+z^2(1+c_1T_{coh})}$

- $A(z) = \frac{-zT_{coh}+z^2T_{coh}}{1+z(T_{coh}(c_2-c_1)-2)+z^2(1+c_1T_{coh})}$

- Perfect aiding $a(z) = \phi(z) \frac{D(z)}{A(z)} = -\phi(z) \frac{1}{T_{coh}} \frac{1-2z+z^2}{z^2-1} = -\phi(z) \frac{1}{T_{coh}} \frac{z-1}{z+1}$

Transfer function from true carrier phase to phase discriminator



Conclusion: phase discriminator contains clock jitter, if no CoOp-aiding is applied

Aided PLL with and without Beamforming

- Standard PLL Transfer Function (no SAP)

- $\hat{\phi}(z) = H(z)\phi(z) + A(z)a(z)$

- $$H(z) = \frac{zT_{coh}(c_2 - c_1) + z^2 T_{coh} c_1}{1 + z(T_{coh}(c_2 - c_1) - 2) + z^2(1 + c_1 T_{coh})}$$
 - $H(z)$... standard 2nd order PLL transfer function
 - $A(z)$... Rate aiding transfer function

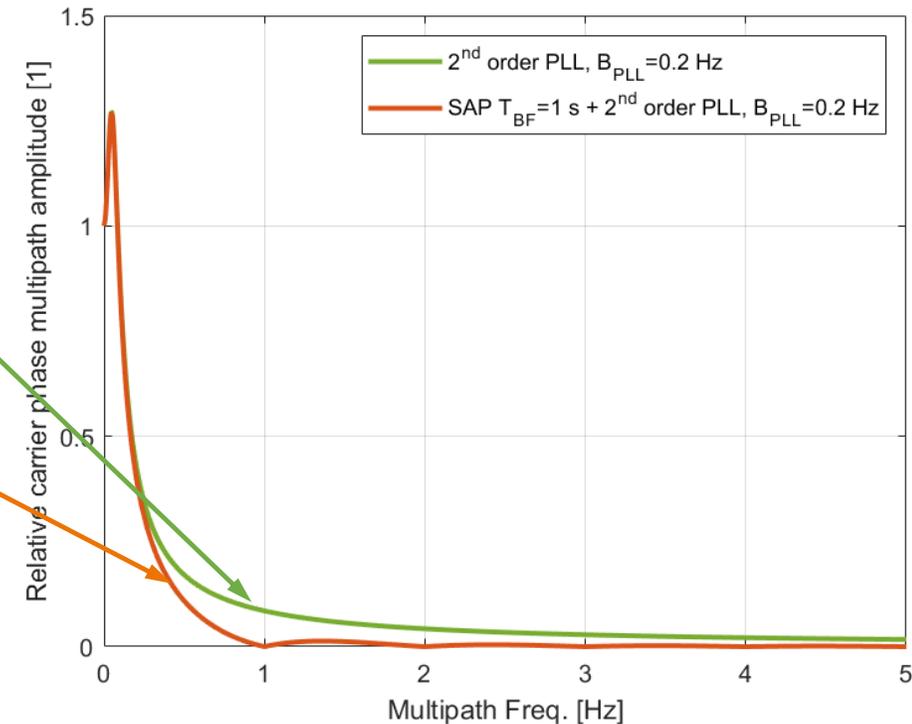
- PLL Transfer Function with SAP

- Assuming perfect aiding, line-of-sight beamforming can be realized as a coherent integration of prompt correlation values
- E.g. realized as moving averaging
- $$H_{SAP,LOS}(z) = \frac{1}{K} \sum_{k=1}^K z^{-k}$$
- Beam-forming time $T_{BF} = KT_{coh}$
- PLL + SAP transfer function $\rightarrow H_{SAP,LOS}(z) H(z)$

- Further Remarks

- Beamforming coefficients have the potential to be tuned, nulling etc.
- No mitigation capability in static situations
 - at least for MEO-GNSS
 - LEO-PNT will exhibit higher multipath frequency even in static situations

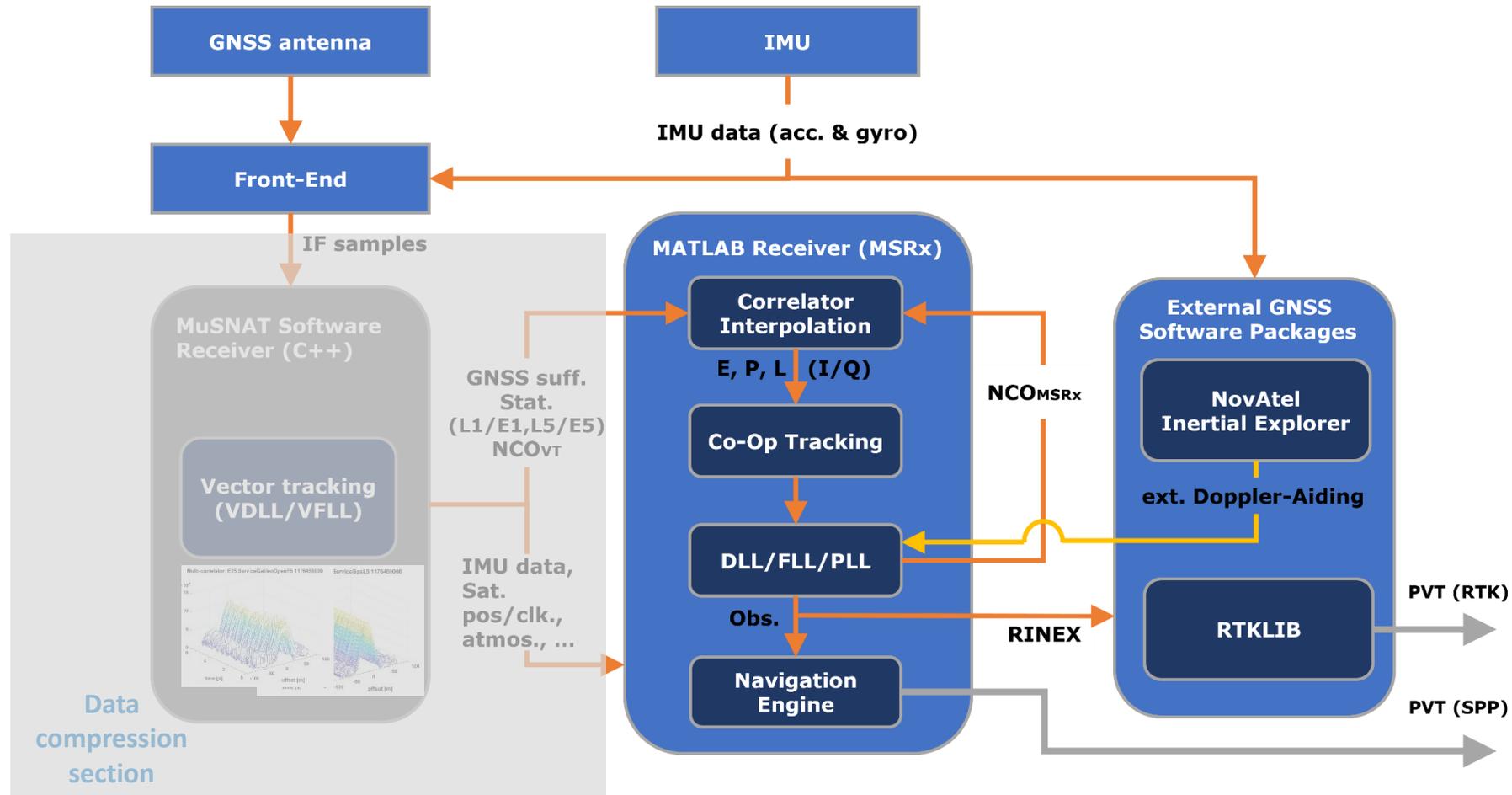
Transfer function from true carrier phase to estimated carrier phase



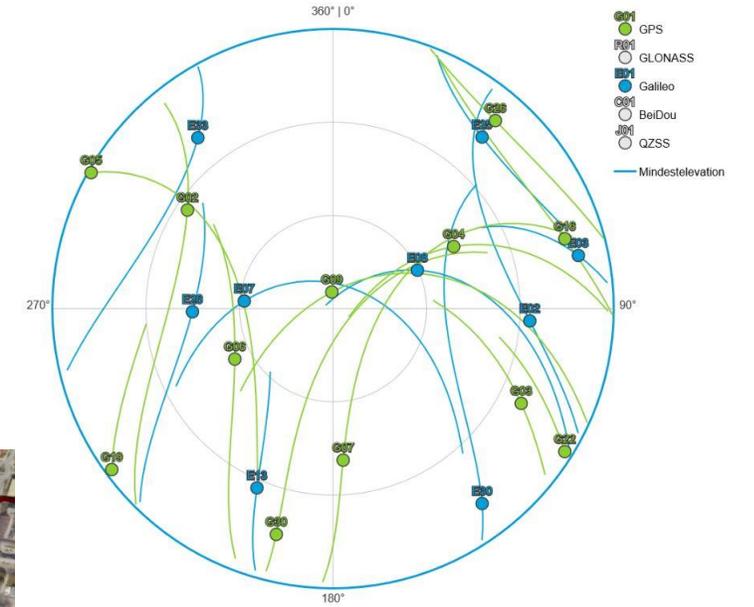
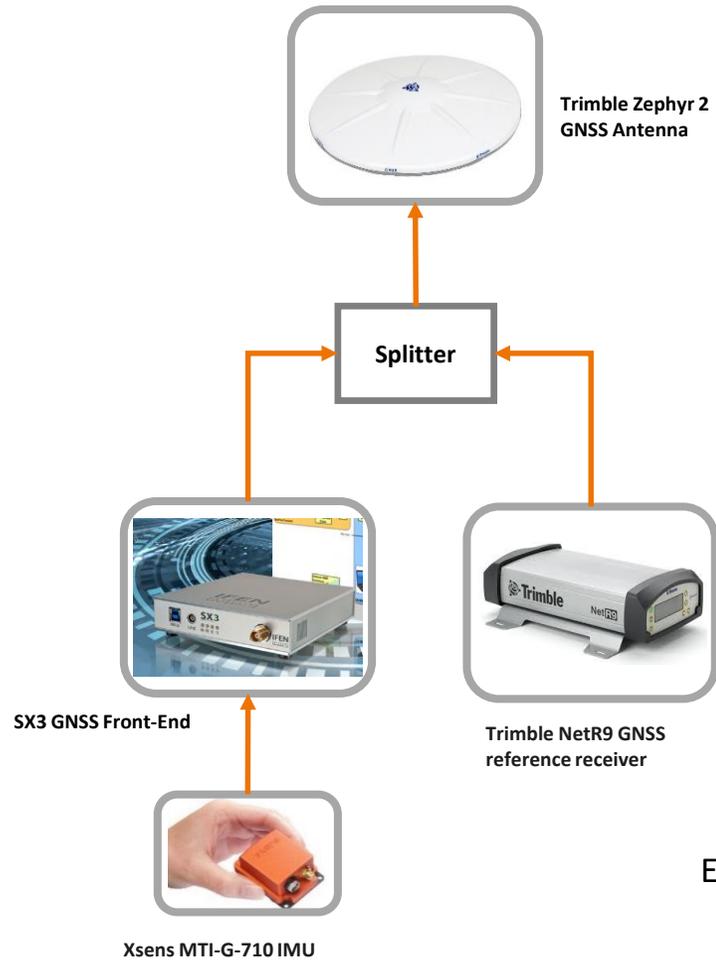
Conclusion: Aided PLL suppress multipath in dynamic situations, SAP will further improve multipath mitigation

Test Implementation MSRx

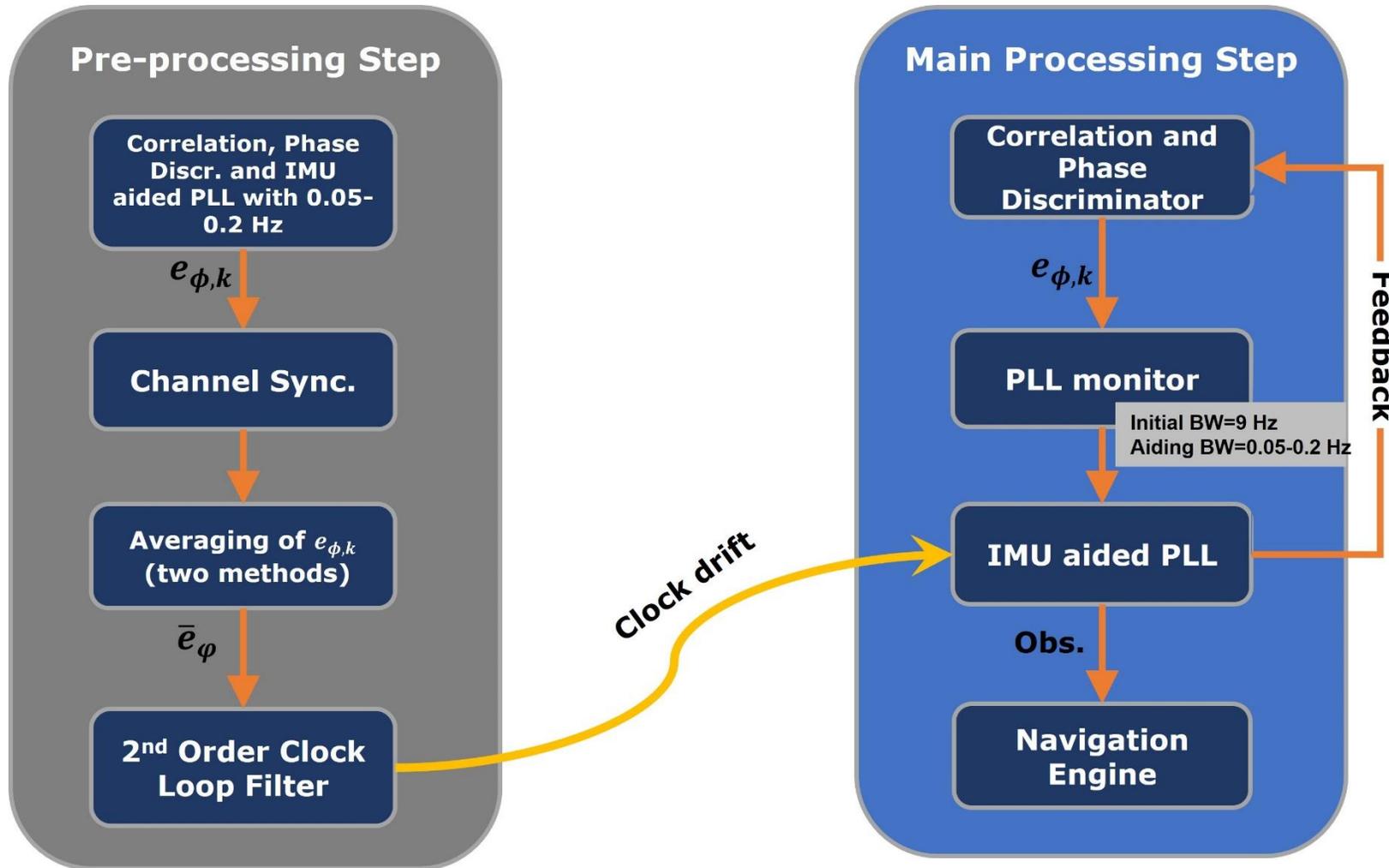
- MATLAB based GNSS SDR using compressed signals as input (Bochkati et al., 2022)
- Supported frequencies: GPS L1 C/A (with data bit wipe-off), L5Q & Galileo E1C, E5aQ



Test Scenario - Drive on UniBw M Campus



CoOp-Estimation – Block Diagram



CoOp-Estimation – Procedure

- Pre-processing step:

1. Synchronization of the PLL tracking channels
 - PLL working with IMU aiding at low bandwidth (0.05-0.2 Hz) $\rightarrow e_{\varphi,k}$ will contain clock jitter
2. Build mean value of phase discriminator from all available channels
 - Method A: Mean of unwrapped phase discriminator
 - Method B: dynamically aligned prompt correlator
3. Apply second order clock loop filter (CLL) \rightarrow similar to 2order conventional PLL and DLL loop filter syntax

$$\begin{bmatrix} \varphi_{k+1} \\ \theta_{k+1} \end{bmatrix} = \begin{bmatrix} 1 & t_{coh} \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \varphi_k \\ \theta_k \end{bmatrix} + \begin{bmatrix} C_2 \\ C_1 \end{bmatrix} \cdot e_{\varphi,k} + \begin{bmatrix} t_{coh} \\ 0 \end{bmatrix} \cdot \delta \dot{t}_{coOp,k}$$

Where $C_1 = \sqrt{2} \cdot t_{coh} \cdot \omega_0$, $\omega_0 = 1.89 \cdot PLL_{BW}$ and $\delta \dot{t}_{coOp,k}$ is the CoOP clock feedback inserted as carrier rates. θ_k represent the linear feedback register

4. Store of the CLL rates for the main processing step run

- Main processing step:

1. The CLL rates are applied directedly to the PLL in the same as the IMU-based Doppler-Aiding
2. Monitoring of PLL necessary, as instable tracking behavior can occur (if occurs, reset PLL to initial high BW of 9 Hz)

Method A: Mean of Unwrapped PLL Discriminator

- k ... channel index
- λ_k ... carrier wavelength in channel k
- $e_{\phi,k}(t)$... PLL phase discriminator in pre-processing step of channel k and at epoch t in [cyc]
- $\overline{e_{\phi}}(t)$... Clock lock loop (CLL) discriminator in [m]

1) Unwrap phase discriminator time series

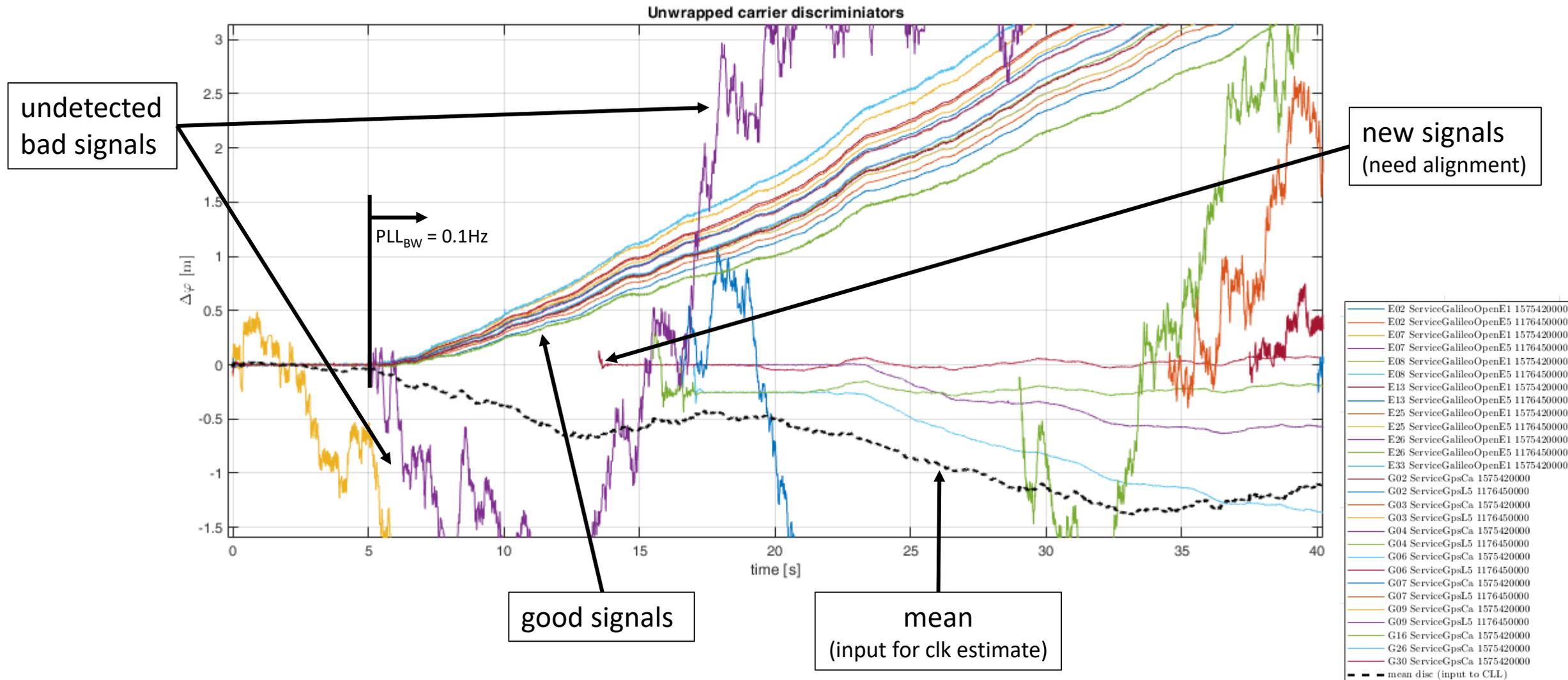
2) Sync. all phase discriminator values to common epoch (via interpolation)

$$e_{\phi,k;unwrap+sync}(t) = \text{unwrap}\{e_{\phi,k}(t)\}$$

3) Averaging

$$\overline{e_{\phi}}(t) = \frac{1}{K} \sum_{k=1}^K \lambda_k e_{\phi,k;unwrap+sync}(t)$$

Method A: Mean of Unwrapped PLL Discriminator



Method B: Dynamically Aligned Prompt Correlator

- k ... channel index
- λ_k ... carrier wavelength in channel k
- $e_{\phi,k}(t)$... phase discriminator in pre-processing step of channel k and at epoch t
- $\varphi(t)$... clock lock loop (CLL) phase in [m]
- $\theta(t)$... CLL rate in [m/s]
- \arg ... argument (i.e. angle) of complex number
- \bar{x} ... complex conjugate of x
- $\bar{e}_{\phi}(t)$... CLL discriminator in [m]

1) Convert discriminator to prompt correlator value $P_k(t) = e^{2\pi j e_{\phi,k}(t)}$

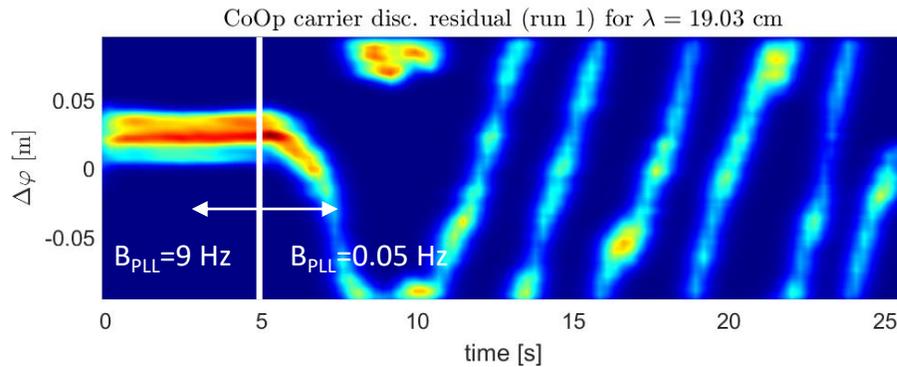
2) Sync. all prompt correlator values to common epoch (via interpolation)

3) Define reference prompt value at beginning of processing (or after outlier detection): $P_{k,off} = P_k(t_0)e^{-2\pi j \varphi(t_0)/\lambda_k}$

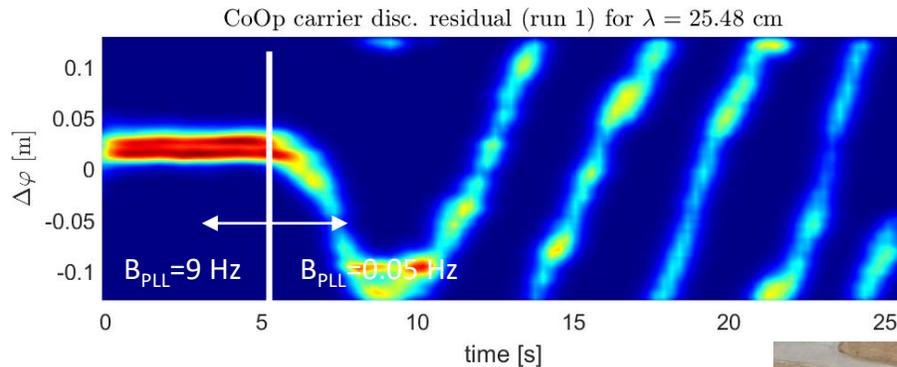
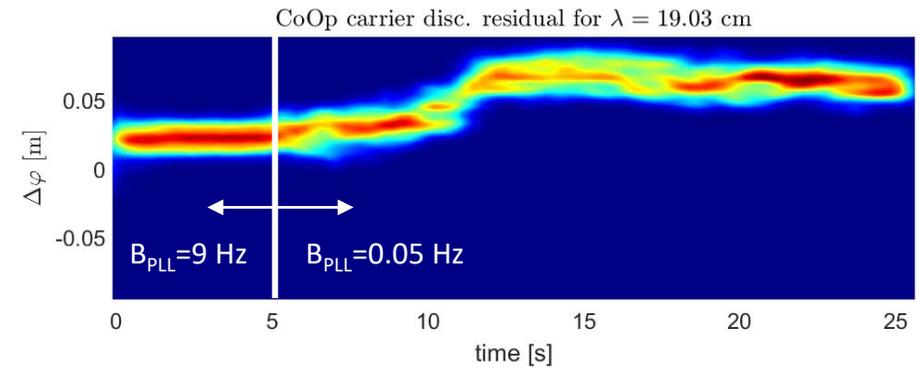
4) Averaging: $\bar{e}_{\phi}(t) = \frac{1}{K} \sum_{k=1}^K \lambda_k \arg\{\bar{P}_{k,off} \cdot P_k(t)e^{-2\pi j \varphi(t)/\lambda_k}\}$

CoOp-Results -Method B: Dynamically Aligned Prompt Correlator

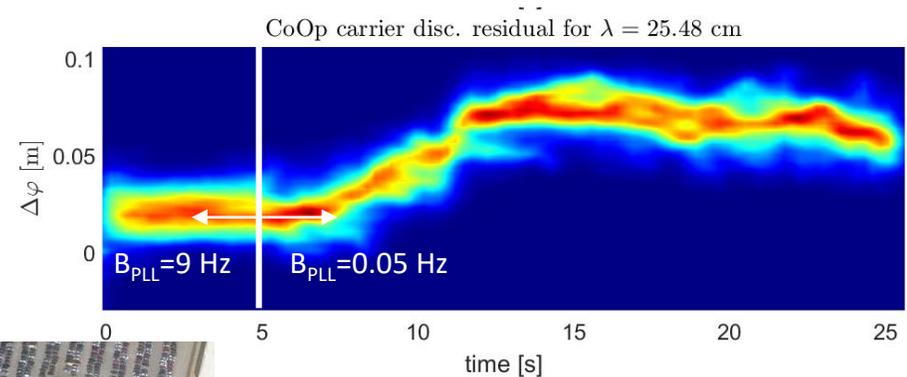
Histogram of phase discriminator values for all channels (separated for L1/E1 and L5/E5a)
 $t_0=383750$ s (car moving, see lower right corner)



L1/E1



L5/E5a

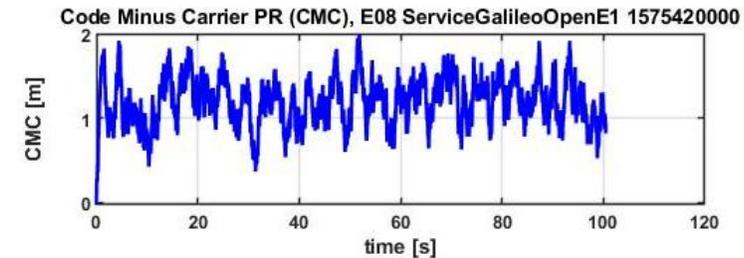
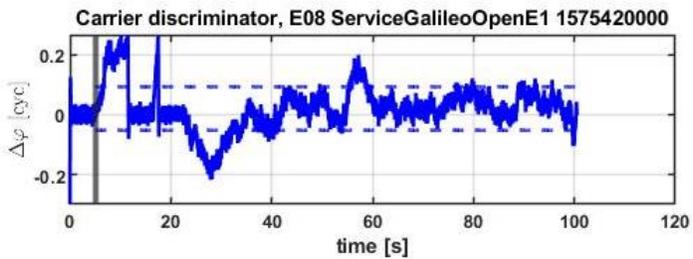
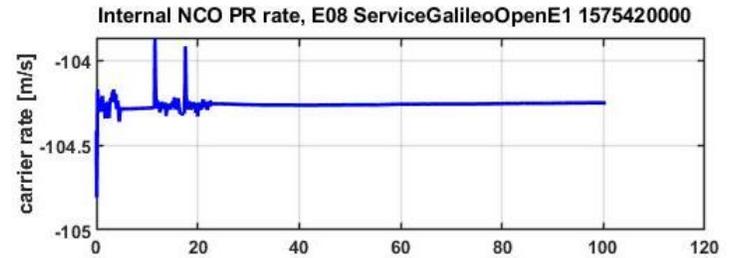
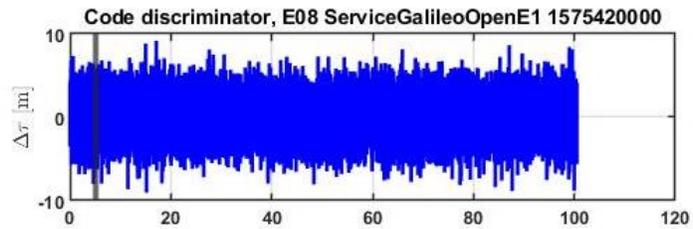
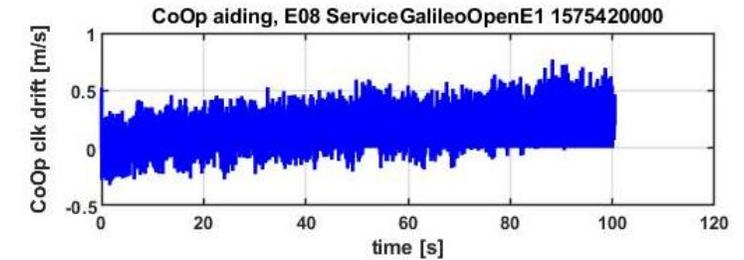
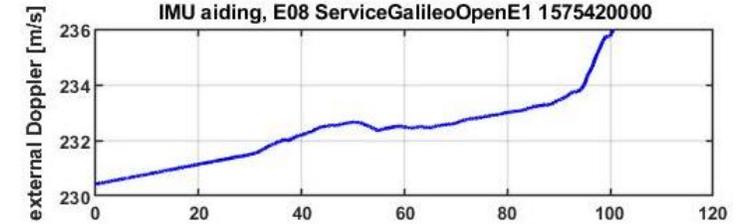
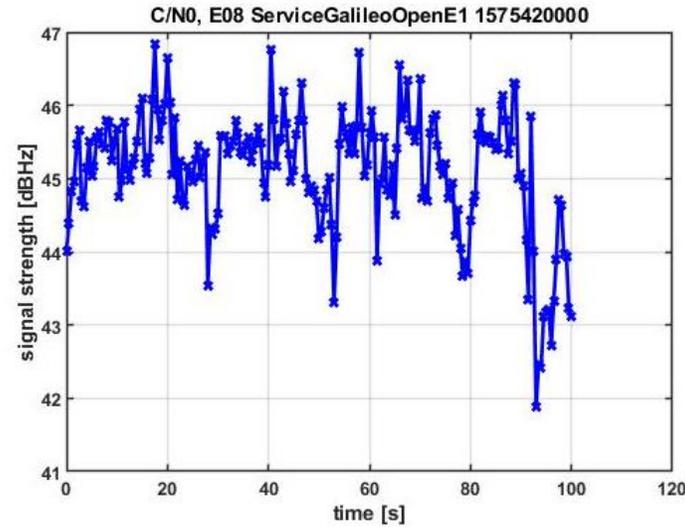
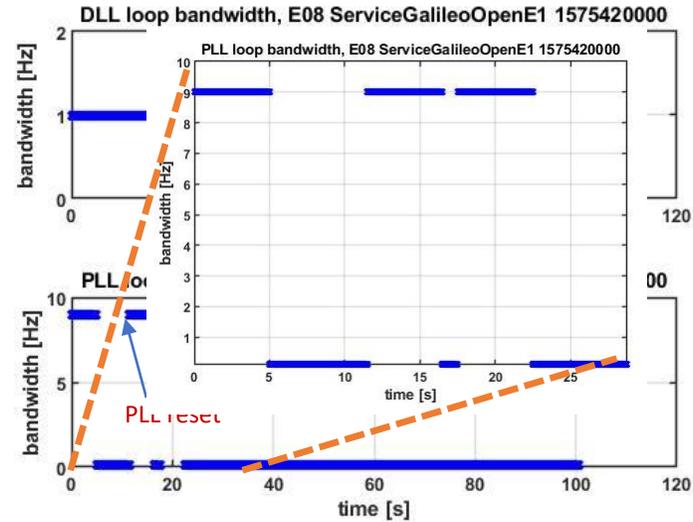


Without CoOp-Aiding (pre-processing step)



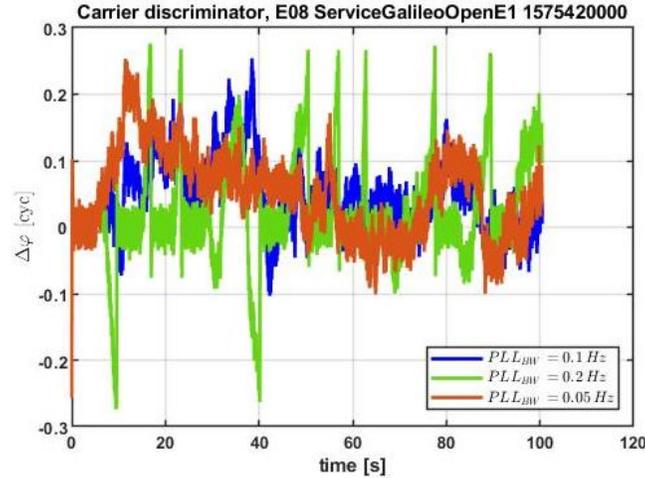
With CoOp-Aiding (main processing step)

Inside the Ultra-Low-Bandwidth PLL Impact of Bandwidth (with CoOp-Aiding)



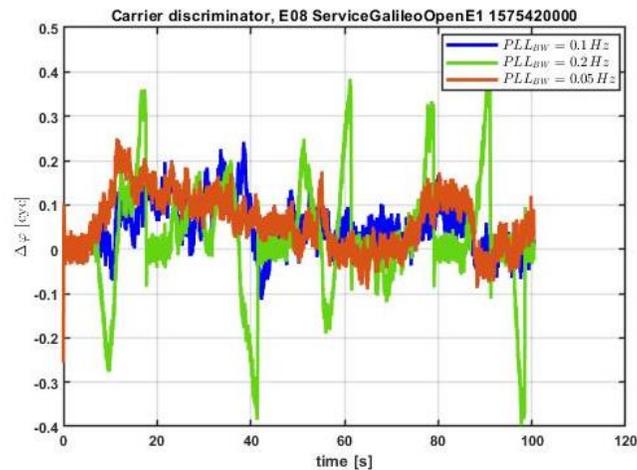
Impact of Bandwidth (with CoOp-Aiding)

- Galileo E1C



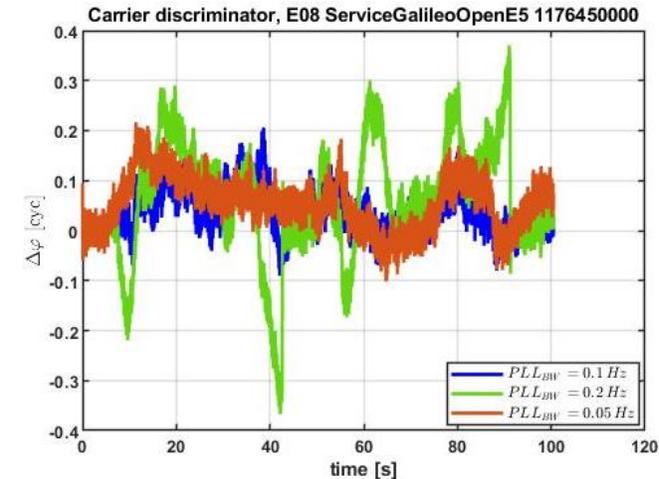
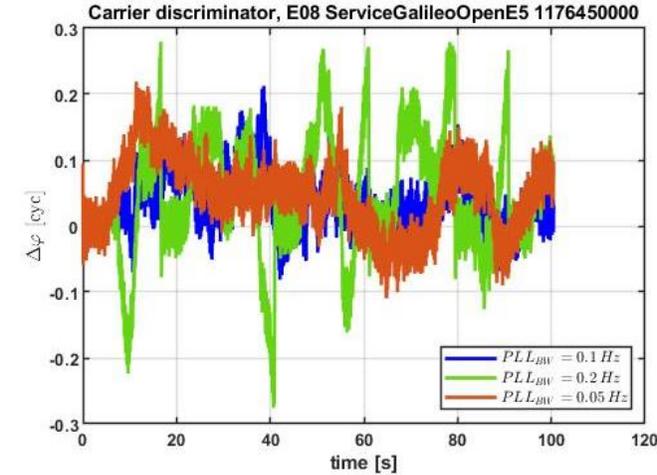
CLL-threshold = 15

CLL_{thres} ... PLL discriminator threshold applied in low bandwidth PLL mode (0.05-0.2 Hz); if exceeded -> reset PLL to initial bandwidth of 9 Hz



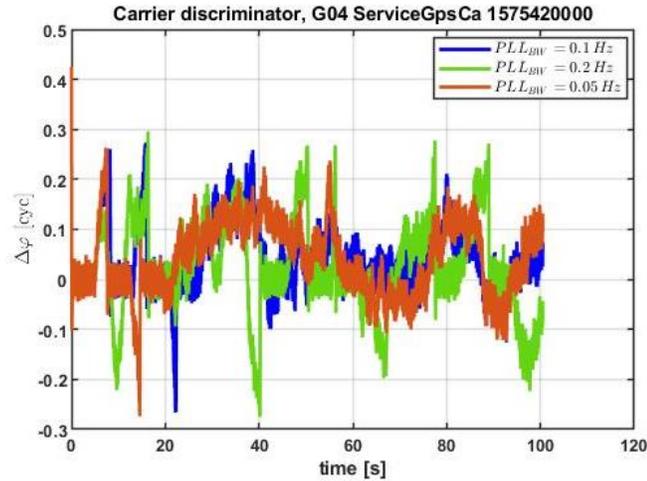
CLL-threshold = 999

- Galileo E5aQ



Impact of Bandwidth (with CoOp-Aiding)

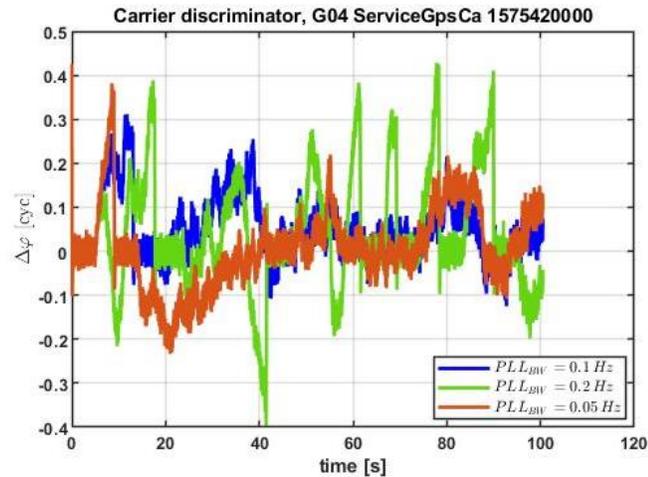
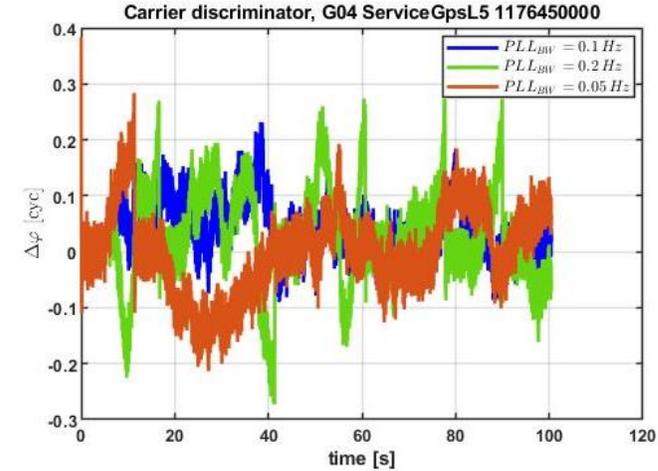
- GPS C/A



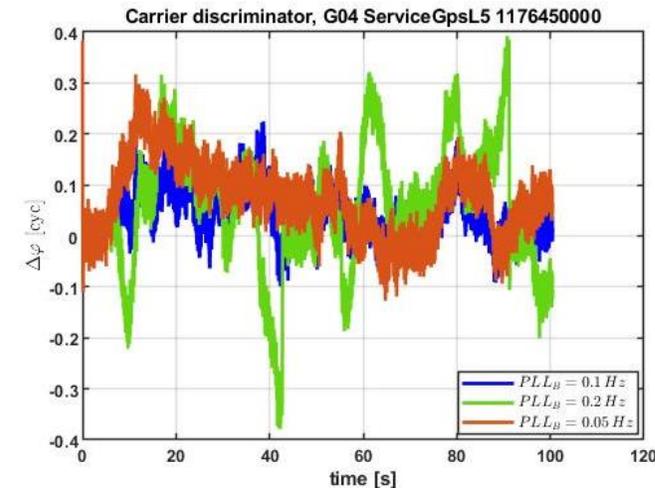
CLL-threshold = 15

CLL_{thres} ... PLL discriminator threshold applied in low bandwidth PLL mode (0.05-0.2 Hz); if exceeded -> reset PLL to initial bandwidth of 9 Hz

- GPS L5Q



CLL-threshold = 999



RTK Ambiguity Fixing Performance

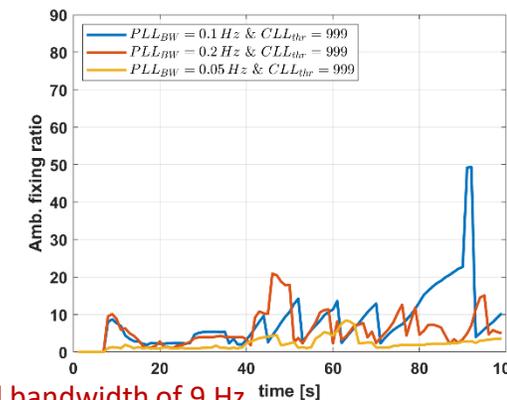
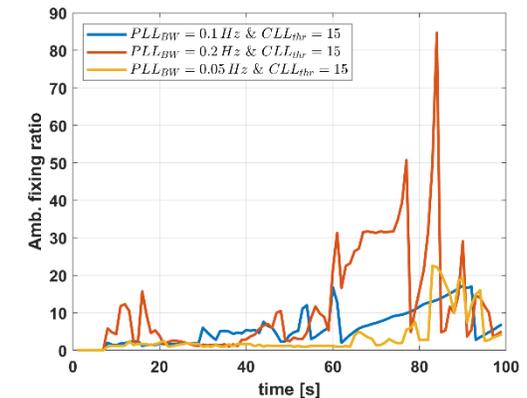
- Post-processing with the open-source GNSS software package „RTKLIB“
- Following RTKLIB-setting have been adapted for all scenarios to process the input RINEX-file

Parameter	Selected settings
RTKLIB version	V2.4.3 b34g (Demo5) GUI
Positioning mode	kinematic
Frequencies	L1/E1C and L5/E5aQ
Satellite system	GPS & Galileo
KF-processing direction	forward
Ambiguity resolution method	continuous
Ambiguity resolution threshold	3

RTK Ambiguity Fixing Performance

- Processing strategies:
 - 3 X MSR_x – RINEX files with $PLL_{BW} = 0.05, 0.1$ and 0.2 Hz & $CLL_{thres} = 15$
 - 3 X MSR_x – RINEX files with $PLL_{BW} = 0.05, 0.1$ and 0.2 Hz & $CLL_{thres} = 999$
- Performance parameters: Fixing rates and Fixing ratio

Scenario	Ambiguity fixing rates [%]
$PLL_{BW} = 0.05$ Hz & $CLL_{thres} = 15$	34.3
$PLL_{BW} = 0.1$ Hz & $CLL_{thres} = 15$	80.8
$PLL_{BW} = 0.2$ Hz & $CLL_{thres} = 15$	78.8
$PLL_{BW} = 0.05$ Hz & $CLL_{thres} = 999$	46.5
$PLL_{BW} = 0.1$ Hz & $CLL_{thres} = 999$	92.9
$PLL_{BW} = 0.2$ Hz & $CLL_{thres} = 999$	83.8



CLL_{thres} ... PLL discriminator threshold applied in low bandwidth PLL mode (0.05-0.2 Hz); if exceeded -> reset PLL to initial bandwidth of 9 Hz

Conclusions / Future Work

- Conclusions

- Analyzed aided PLL and synthetic aperture processing with z-transform
- Prototype CoOp L1/E1/E5/L5 implementation working with dynamic real-world data, ultra-low bandwidth PLL demonstrated
- Solution RTK-capable, but further tuning required to achieve 100 % in open sky

- Future Work:

- Develop deeper understanding of error budget (transient errors due to slightly incorrect aiding apparently relevant)
- Test the performance of Doppler-aided PLL + CoOp oscillator tracking in urban canopies.
- Elaborate deeper understanding for the trade-off between the aided PLL bandwidth and the CoOp bandwidth
- Implementation of the SAP algorithms to further improve and support the tracking capability of the MSRx receiver
- Replace CoOp Method “Aligned Prompt Correlator” by Kalman Filter (including data screening, CN0 threshold, etc.)



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