# Arithmetic Expressions

```
options
   start_node_filter = "x";
end

grammar Expr
   nonterminal  E(2), T(2), F(2), S(0);
   terminal     plus(2), mult(2), open(2), close(2), number(2);
   start        S;

   S()          ::= E(x,y)                          [ init   ]
   E(x,y)       ::= T(x,y)                          [ expr1  ]
                 |  E(x,u) plus(u,v) T(v,y)         [ expr2  ]
   T(x,y)       ::= F(x,y)                          [ term1  ]
                 |  T(x,u) mult(u,v) F(v,y)         [ term2  ]
   F(x,y)       ::= open(x,u) E(u,v) close(v,y)     [ paren  ]
                 |  number(x,y)                     [ number ]
end
```

**State $q_0(\boldsymbol{a})$**

| | |
|---|---|
| $\mathsf{S}()$ | $\to \centerdot\, \mathsf{E}(\boldsymbol{a}, n_1)$ |
| $\mathsf{E}(\boldsymbol{a}, n_2)$ | $\to \centerdot\, \mathsf{E}(\boldsymbol{a}, n_3)\ \mathsf{plus}(n_3, n_4)\ \mathsf{T}(n_4, n_2)$ |
| $\mathsf{E}(\boldsymbol{a}, n_5)$ | $\to \centerdot\, \mathsf{T}(\boldsymbol{a}, n_5)$ |
| $\mathsf{F}(\boldsymbol{a}, n_6)$ | $\to \centerdot\, \mathsf{number}(\boldsymbol{a}, n_6)$ |
| $\mathsf{F}(\boldsymbol{a}, n_7)$ | $\to \centerdot\, \mathsf{open}(\boldsymbol{a}, n_8)\ \mathsf{E}(n_8, n_9)\ \mathsf{close}(n_9, n_7)$ |
| $\mathsf{T}(\boldsymbol{a}, n_{10})$ | $\to \centerdot\, \mathsf{F}(\boldsymbol{a}, n_{10})$ |
| $\mathsf{T}(\boldsymbol{a}, n_{11})$ | $\to \centerdot\, \mathsf{T}(\boldsymbol{a}, n_{12})\ \mathsf{mult}(n_{12}, n_{13})\ \mathsf{F}(n_{13}, n_{11})$ |

$$\xrightarrow[n_0 = \boldsymbol{a}, n_1\uparrow]{\mathsf{E}(n_0, n_1)} q_7(n_0, n_1)$$
$$\xrightarrow[n_0 = \boldsymbol{a}, n_1\uparrow]{\mathsf{F}(n_0, n_1)} q_3(n_0, n_1)$$
$$\xrightarrow[n_0 = \boldsymbol{a}, n_1\uparrow]{\mathsf{T}(n_0, n_1)} q_4(n_0, n_1)$$
$$\xrightarrow[n_0 = \boldsymbol{a}, n_1\uparrow]{\mathsf{number}(n_0, n_1)} q_1(n_0, n_1)$$
$$\xrightarrow[n_0 = \boldsymbol{a}, n_1\uparrow]{\mathsf{open}(n_0, n_1)} q_2(n_0, n_1)$$

**State $q_1(\boldsymbol{a}, \boldsymbol{b})$**

| | |
|---|---|
| $\mathsf{F}(\boldsymbol{a}, \boldsymbol{b}) \to \mathsf{number}(\boldsymbol{a}, \boldsymbol{b})\,\centerdot$ | $[\mathit{number}]$ |

**State $q_2(\boldsymbol{a}, \boldsymbol{b})$**

| | |
|---|---|
| $\mathsf{F}(\boldsymbol{a}, n_1)$ | $\to \mathsf{open}(\boldsymbol{a}, \boldsymbol{b})\ \centerdot\ \mathsf{E}(\boldsymbol{b}, n_2)\ \mathsf{close}(n_2, n_1)$ |
| $\mathsf{E}(\boldsymbol{b}, n_3)$ | $\to \centerdot\, \mathsf{E}(\boldsymbol{b}, n_4)\ \mathsf{plus}(n_4, n_5)\ \mathsf{T}(n_5, n_3)$ |
| $\mathsf{E}(\boldsymbol{b}, n_6)$ | $\to \centerdot\, \mathsf{T}(\boldsymbol{b}, n_6)$ |
| $\mathsf{F}(\boldsymbol{b}, n_7)$ | $\to \centerdot\, \mathsf{number}(\boldsymbol{b}, n_7)$ |
| $\mathsf{F}(\boldsymbol{b}, n_8)$ | $\to \centerdot\, \mathsf{open}(\boldsymbol{b}, n_9)\ \mathsf{E}(n_9, n_{10})\ \mathsf{close}(n_{10}, n_8)$ |
| $\mathsf{T}(\boldsymbol{b}, n_{11})$ | $\to \centerdot\, \mathsf{F}(\boldsymbol{b}, n_{11})$ |
| $\mathsf{T}(\boldsymbol{b}, n_{12})$ | $\to \centerdot\, \mathsf{T}(\boldsymbol{b}, n_{13})\ \mathsf{mult}(n_{13}, n_{14})\ \mathsf{F}(n_{14}, n_{12})$ |

$$\xrightarrow[n_0 = \boldsymbol{b}, n_1\uparrow]{\mathsf{E}(n_0, n_1)} q_8(n_0, n_1, \boldsymbol{a})$$
$$\xrightarrow[n_0 = \boldsymbol{b}, n_1\uparrow]{\mathsf{F}(n_0, n_1)} q_3(n_0, n_1)$$
$$\xrightarrow[n_0 = \boldsymbol{b}, n_1\uparrow]{\mathsf{T}(n_0, n_1)} q_4(n_0, n_1)$$
$$\xrightarrow[n_0 = \boldsymbol{b}, n_1\uparrow]{\mathsf{number}(n_0, n_1)} q_1(n_0, n_1)$$
$$\xrightarrow[n_0 = \boldsymbol{b}, n_1\uparrow]{\mathsf{open}(n_0, n_1)} q_2(n_0, n_1)$$

**State $q_3(\boldsymbol{a}, \boldsymbol{b})$**

| | |
|---|---|
| $\mathsf{T}(\boldsymbol{a}, \boldsymbol{b}) \to \mathsf{F}(\boldsymbol{a}, \boldsymbol{b})\,\centerdot$ | $[\mathit{term1}]$ |

**State $\boldsymbol{q_4}(\boldsymbol{a},\boldsymbol{b})$**

$$\begin{array}{|lll|}\hline \mathsf{E}(\boldsymbol{a},\boldsymbol{b}) & \to \mathsf{T}(\boldsymbol{a},\boldsymbol{b})\,\bullet & [\mathit{expr1}] \\ \mathsf{T}(\boldsymbol{a},n_1) & \to \mathsf{T}(\boldsymbol{a},\boldsymbol{b})\,\bullet\,\mathsf{mult}(\boldsymbol{b},n_2)\,\mathsf{F}(n_2,n_1) & \\ \hline \end{array}$$

$$\xrightarrow[n_0=\boldsymbol{b},\,n_1\uparrow]{\mathsf{mult}(n_0,n_1)} q_5(\boldsymbol{a},n_0,n_1)$$

**State $\boldsymbol{q_5}(\boldsymbol{a},\boldsymbol{b},\boldsymbol{c})$**

$$\begin{array}{|ll|}\hline \mathsf{T}(\boldsymbol{a},n_1) & \to \mathsf{T}(\boldsymbol{a},\boldsymbol{b})\,\mathsf{mult}(\boldsymbol{b},\boldsymbol{c})\,\bullet\,\mathsf{F}(\boldsymbol{c},n_1) \\ \hline \mathsf{F}(\boldsymbol{c},n_2) & \to \bullet\,\mathsf{number}(\boldsymbol{c},n_2) \\ \mathsf{F}(\boldsymbol{c},n_3) & \to \bullet\,\mathsf{open}(\boldsymbol{c},n_4)\,\mathsf{E}(n_4,n_5)\,\mathsf{close}(n_5,n_3) \\ \hline \end{array}$$

$$\xrightarrow[n_0=\boldsymbol{c},\,n_1\uparrow]{\mathsf{F}(n_0,n_1)} q_6(\boldsymbol{a},n_1,\boldsymbol{b},n_0)$$
$$\xrightarrow[n_0=\boldsymbol{c},\,n_1\uparrow]{\mathsf{number}(n_0,n_1)} q_1(n_0,n_1)$$
$$\xrightarrow[n_0=\boldsymbol{c},\,n_1\uparrow]{\mathsf{open}(n_0,n_1)} q_2(n_0,n_1)$$

**State $\boldsymbol{q_6}(\boldsymbol{a},\boldsymbol{b},\boldsymbol{c},\boldsymbol{d})$**

$$\begin{array}{|lll|}\hline \mathsf{T}(\boldsymbol{a},\boldsymbol{b}) & \to \mathsf{T}(\boldsymbol{a},\boldsymbol{c})\,\mathsf{mult}(\boldsymbol{c},\boldsymbol{d})\,\mathsf{F}(\boldsymbol{d},\boldsymbol{b})\,\bullet & [\mathit{term2}] \\ \hline \end{array}$$

**State $\boldsymbol{q_7}(\boldsymbol{a},\boldsymbol{b})$**

$$\begin{array}{|lll|}\hline \mathsf{E}(\boldsymbol{a},n_1) & \to \mathsf{E}(\boldsymbol{a},\boldsymbol{b})\,\bullet\,\mathsf{plus}(\boldsymbol{b},n_2)\,\mathsf{T}(n_2,n_1) & \\ \mathsf{S}() & \to \mathsf{E}(\boldsymbol{a},\boldsymbol{b})\,\bullet & [\mathit{init}] \\ \hline \end{array}$$

$$\xrightarrow[n_0=\boldsymbol{b},\,n_1\uparrow]{\mathsf{plus}(n_0,n_1)} q_9(\boldsymbol{a},n_0,n_1)$$

**State $\boldsymbol{q_8}(\boldsymbol{a},\boldsymbol{b},\boldsymbol{c})$**

$$\begin{array}{|ll|}\hline \mathsf{E}(\boldsymbol{a},n_1) & \to \mathsf{E}(\boldsymbol{a},\boldsymbol{b})\,\bullet\,\mathsf{plus}(\boldsymbol{b},n_2)\,\mathsf{T}(n_2,n_1) \\ \mathsf{F}(\boldsymbol{c},n_3) & \to \mathsf{open}(\boldsymbol{c},\boldsymbol{a})\,\mathsf{E}(\boldsymbol{a},\boldsymbol{b})\,\bullet\,\mathsf{close}(\boldsymbol{b},n_3) \\ \hline \end{array}$$

$$\xrightarrow[n_0=\boldsymbol{b},\,n_1\uparrow]{\mathsf{close}(n_0,n_1)} q_{10}(\boldsymbol{c},n_1,\boldsymbol{a},n_0)$$
$$\xrightarrow[n_0=\boldsymbol{b},\,n_1\uparrow]{\mathsf{plus}(n_0,n_1)} q_9(\boldsymbol{a},n_0,n_1)$$

**State $\boldsymbol{q_9}(\boldsymbol{a},\boldsymbol{b},\boldsymbol{c})$**

$$\begin{array}{|ll|}\hline \mathsf{E}(\boldsymbol{a},n_1) & \to \mathsf{E}(\boldsymbol{a},\boldsymbol{b})\,\mathsf{plus}(\boldsymbol{b},\boldsymbol{c})\,\bullet\,\mathsf{T}(\boldsymbol{c},n_1) \\ \hline \mathsf{F}(\boldsymbol{c},n_2) & \to \bullet\,\mathsf{number}(\boldsymbol{c},n_2) \\ \mathsf{F}(\boldsymbol{c},n_3) & \to \bullet\,\mathsf{open}(\boldsymbol{c},n_4)\,\mathsf{E}(n_4,n_5)\,\mathsf{close}(n_5,n_3) \\ \mathsf{T}(\boldsymbol{c},n_6) & \to \bullet\,\mathsf{F}(\boldsymbol{c},n_6) \\ \mathsf{T}(\boldsymbol{c},n_7) & \to \bullet\,\mathsf{T}(\boldsymbol{c},n_8)\,\mathsf{mult}(n_8,n_9)\,\mathsf{F}(n_9,n_7) \\ \hline \end{array}$$

$$\xrightarrow[n_0=\boldsymbol{c},\,n_1\uparrow]{\mathsf{F}(n_0,n_1)} q_3(n_0,n_1)$$
$$\xrightarrow[n_0=\boldsymbol{c},\,n_1\uparrow]{\mathsf{T}(n_0,n_1)} q_{11}(n_0,n_1,\boldsymbol{a},\boldsymbol{b})$$
$$\xrightarrow[n_0=\boldsymbol{c},\,n_1\uparrow]{\mathsf{number}(n_0,n_1)} q_1(n_0,n_1)$$
$$\xrightarrow[n_0=\boldsymbol{c},\,n_1\uparrow]{\mathsf{open}(n_0,n_1)} q_2(n_0,n_1)$$

**State $\boldsymbol{q_{10}}(\boldsymbol{a},\boldsymbol{b},\boldsymbol{c},\boldsymbol{d})$**

$$\begin{array}{|lll|}\hline \mathsf{F}(\boldsymbol{a},\boldsymbol{b}) & \to \mathsf{open}(\boldsymbol{a},\boldsymbol{c})\,\mathsf{E}(\boldsymbol{c},\boldsymbol{d})\,\mathsf{close}(\boldsymbol{d},\boldsymbol{b})\,\bullet & [\mathit{paren}] \\ \hline \end{array}$$

**State $\boldsymbol{q_{11}}(\boldsymbol{a},\boldsymbol{b},\boldsymbol{c},\boldsymbol{d})$**

$$\begin{array}{|lll|}\hline \mathsf{T}(\boldsymbol{a},n_1) & \to \mathsf{T}(\boldsymbol{a},\boldsymbol{b})\,\bullet\,\mathsf{mult}(\boldsymbol{b},n_2)\,\mathsf{F}(n_2,n_1) & \\ \mathsf{E}(\boldsymbol{c},\boldsymbol{b}) & \to \mathsf{E}(\boldsymbol{c},\boldsymbol{d})\,\mathsf{plus}(\boldsymbol{d},\boldsymbol{a})\,\mathsf{T}(\boldsymbol{a},\boldsymbol{b})\,\bullet & [\mathit{expr2}] \\ \hline \end{array}$$

$$\xrightarrow[n_0=\boldsymbol{b},\,n_1\uparrow]{\mathsf{mult}(n_0,n_1)} q_5(\boldsymbol{a},n_0,n_1)$$