Mesh Tying and Contact Algorithms for Nonlinear Beam-to-Solid Interactions

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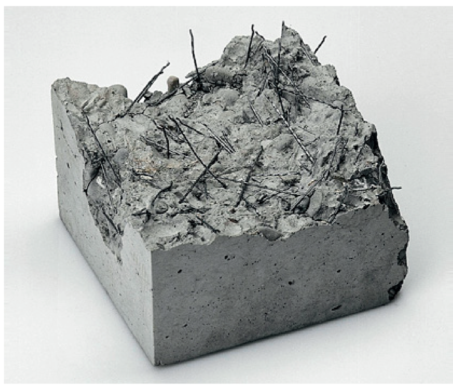
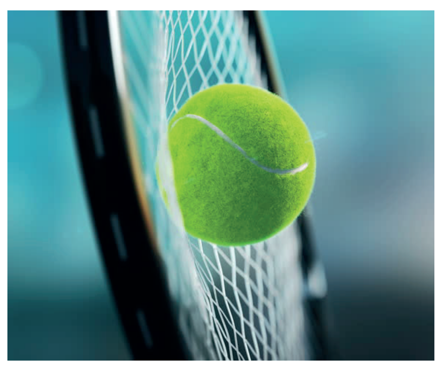
Motivation

The interaction of beam-like structures with three-dimensional objects (1D-3D) can be found in a variety of different applications. Mechanically, the inteaction can be divided into three different types:

Beam-to-Solid ...

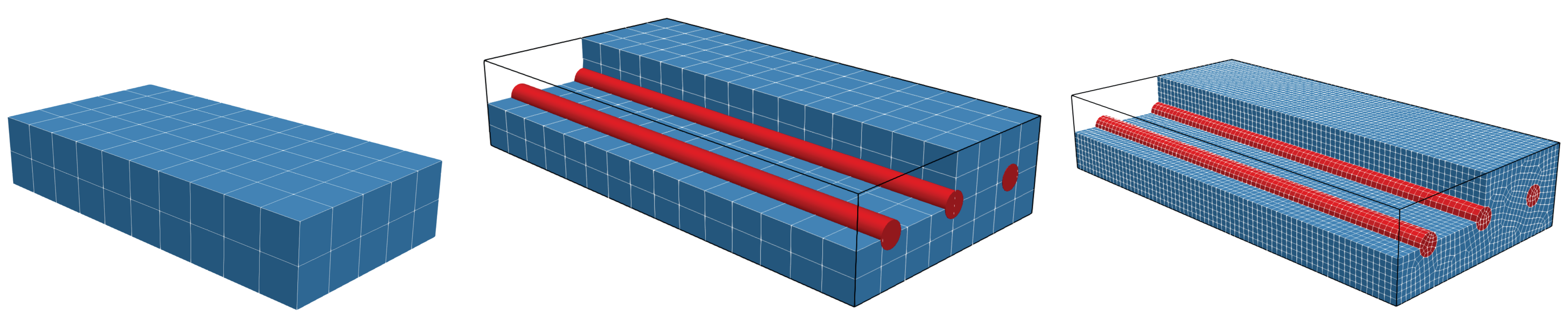
volume coupling	**surface coupling**	**contact**





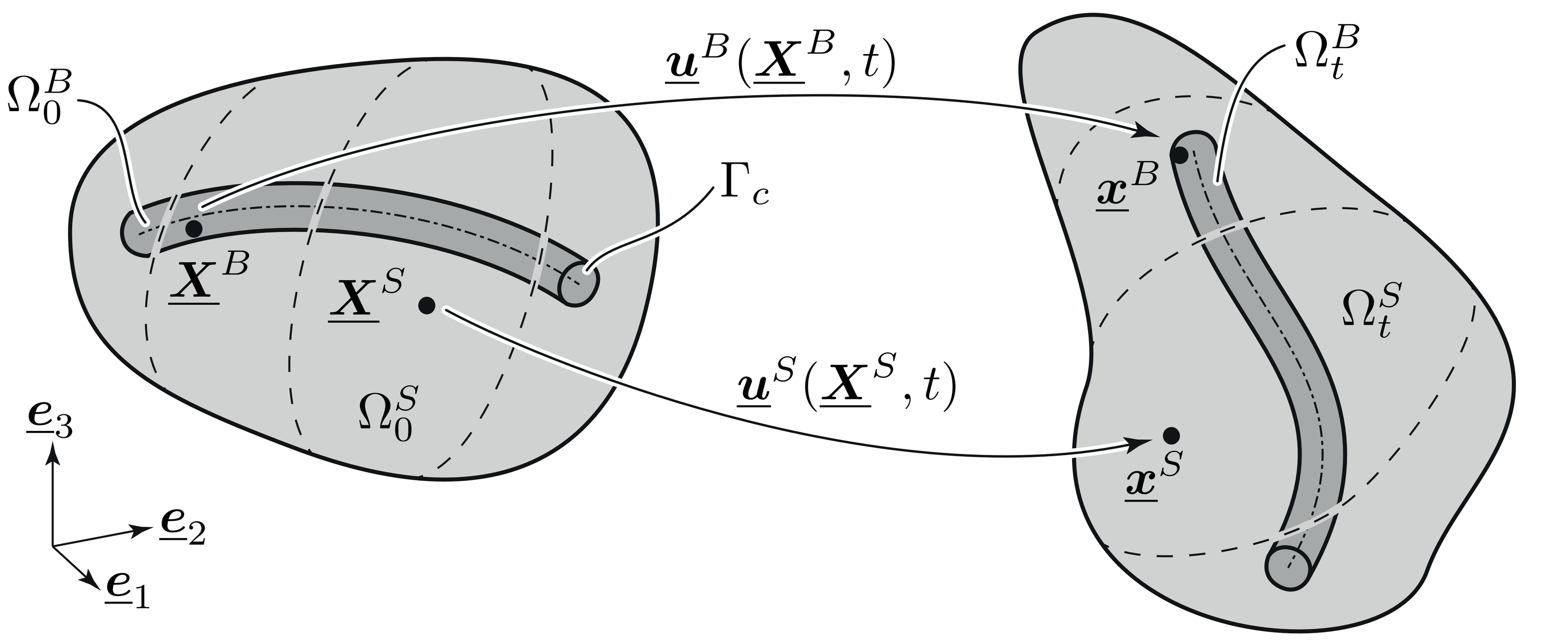
Beam-to-solid volume coupling can be considered the most fundamental of the three cases and will be investigated in detail here.

Different methods exist to model embedded beams in volumes. The proposed approach [1] aims to fill the gap between homogenized and full 3D models by coupling efficient 1D beam elements [2] to 3D solids.



homogenized → **beam-to-solid mesh tying** → 3D model of beam

Problem Description



In the beam-to-solid volume mesh tying case the displacements of the beam surface are coupled to the solid, via

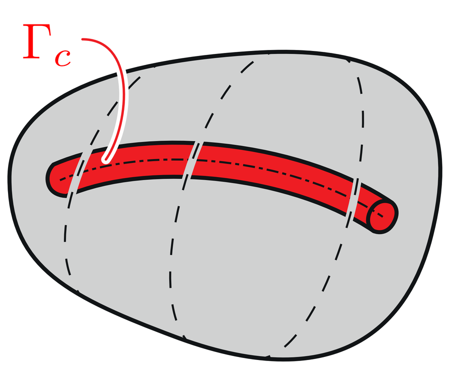
$$\underline{\boldsymbol{u}}^B - \underline{\boldsymbol{u}}^S = \underline{\boldsymbol{0}} \quad \text{on} \quad \Gamma_c.$$

Using a mortar method [3] to discretize the coupling constraints leads to the following coupling contributions to the global weak form, where $\underline{\boldsymbol{\lambda}}$ are the coupling tractions, i.e. the Lagrange multipliers

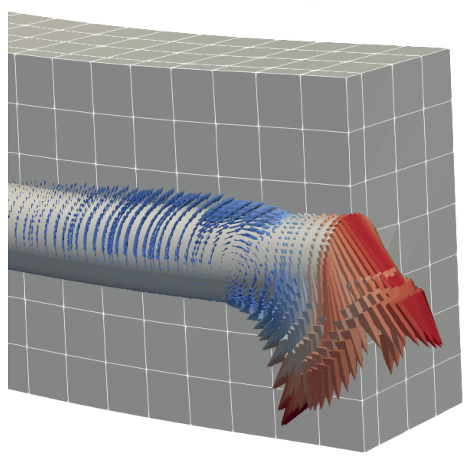
$$-\delta W_c^f = \int_{\Gamma_c} \underline{\boldsymbol{\lambda}} \left(\delta \underline{\boldsymbol{u}}^B - \delta \underline{\boldsymbol{u}}^S\right) \mathrm{d}A_0\,, \quad \delta W_\lambda^f = \int_{\Gamma_c} \delta \underline{\boldsymbol{\lambda}} \left(\underline{\boldsymbol{u}}^B - \underline{\boldsymbol{u}}^S\right) \mathrm{d}A_0\,.$$

The evaluation of the surface integrals is computationally expensive. To circumvent this problem, the integrals over the beam surface are approximated with integrals over the beam centerline. This changes the coupling from a surface-to-volume coupling to a line-to-volume coupling.

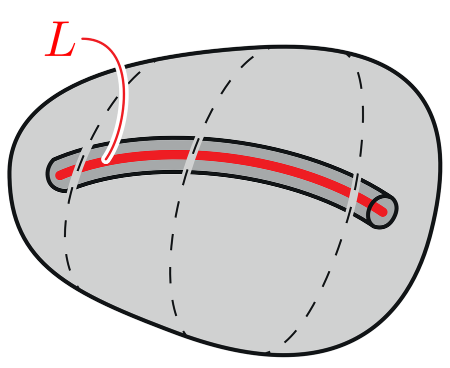
surface-to-volume



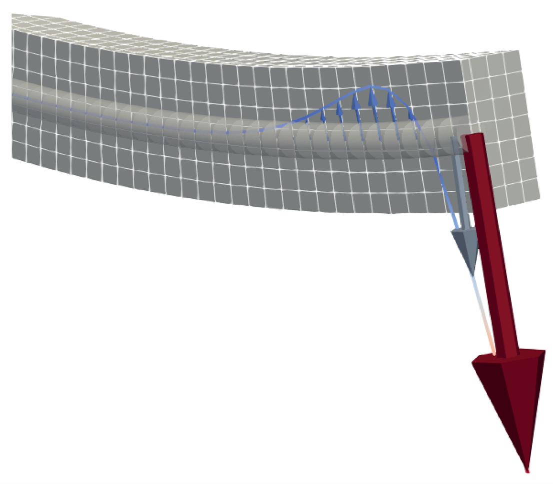
$$= \int_{\Gamma_c} \ldots \mathrm{d}A_0 \;\rightarrow$$



line-to-volume



$$\approx \int_{L} \ldots \mathrm{d}s \;\rightarrow$$



Discretization

The primary field variables are approximated with a finite element discretization

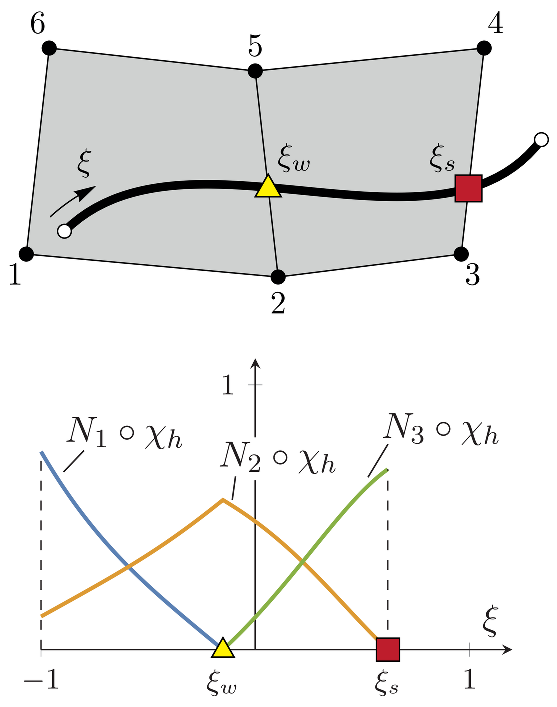
$$\underline{\boldsymbol{u}}_h^S = \sum_{k=1}^{n_S} N_k \boldsymbol{d}^S{}_k \qquad \underline{\boldsymbol{u}}_h^B = \sum_{k=1}^{n_B} H_k \boldsymbol{d}_k^B \qquad \underline{\boldsymbol{\lambda}}_h = \sum_{k=1}^{n_\lambda} \Phi_k \boldsymbol{\lambda}_k$$

The discretized coupling terms follow as

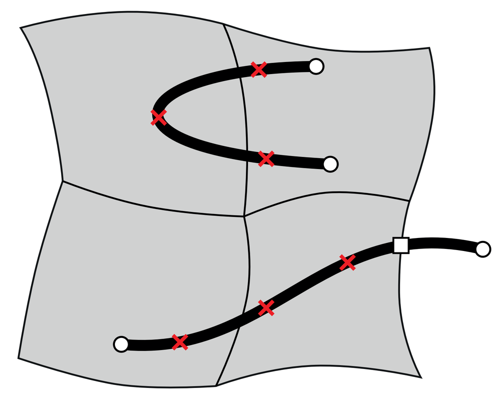
$$-\delta W_{c,h} = \delta \boldsymbol{d}^{B\,\mathrm{T}} \boldsymbol{D}^{\mathrm{T}} \boldsymbol{\lambda} - \delta \boldsymbol{d}^{S\,\mathrm{T}} \boldsymbol{M}^{\mathrm{T}} \boldsymbol{\lambda} \qquad \boldsymbol{D}[j,k] = \int_L \Phi_j H_k \mathrm{d}s\, \boldsymbol{I}$$

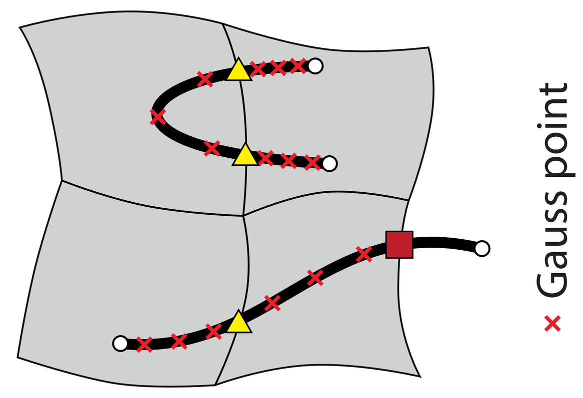
$$\delta W_{\lambda,h} = \delta \boldsymbol{\lambda}^{\mathrm{T}} \boldsymbol{D} \boldsymbol{d}^B - \delta \boldsymbol{\lambda}^{\mathrm{T}} \boldsymbol{M} \boldsymbol{d}^S \quad \text{with} \quad \boldsymbol{M}[j,k] = \int_L \Phi_j \left(N_k \circ \chi_h\right) \mathrm{d}s\, \boldsymbol{I}$$

The discrete mapping $\circ\chi_h$ of the solid shape functions on to the beam results in strong ■ and weak ▲ discontinuities, which has effects on the accuracy of the numerical integration. Two integration algorithms are compared, element- and segment-based integration [4].



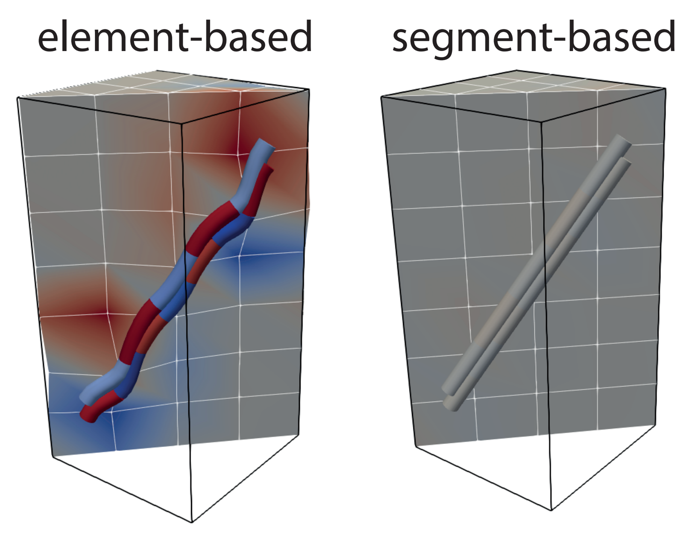
element-based	segment-based
- simple algorithm - low accuracy across discontinuities	- complex algorithm - accurate even with low number of Gauss points



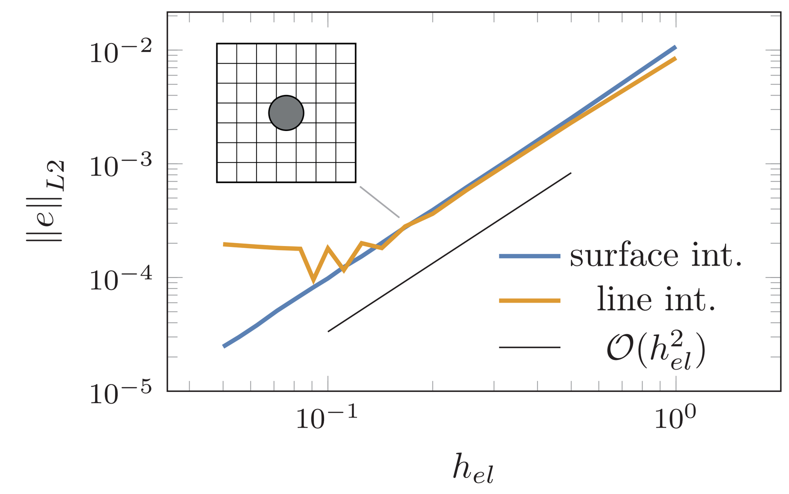


Numerical Examples

consistency / verification

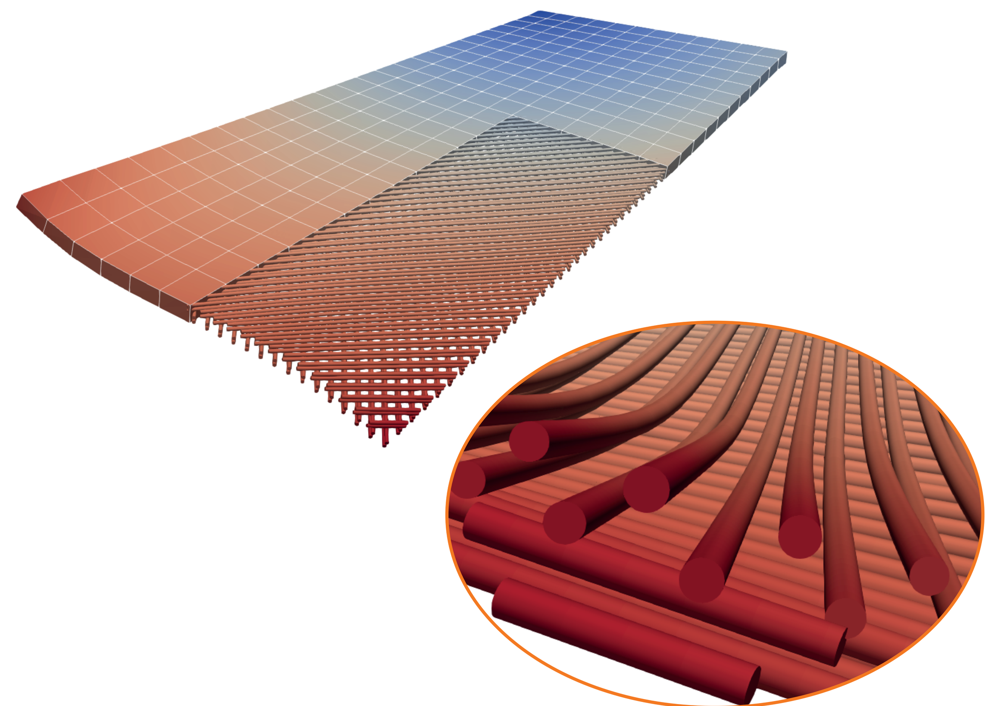


Patch test are only fulfilled for segment-based integration.

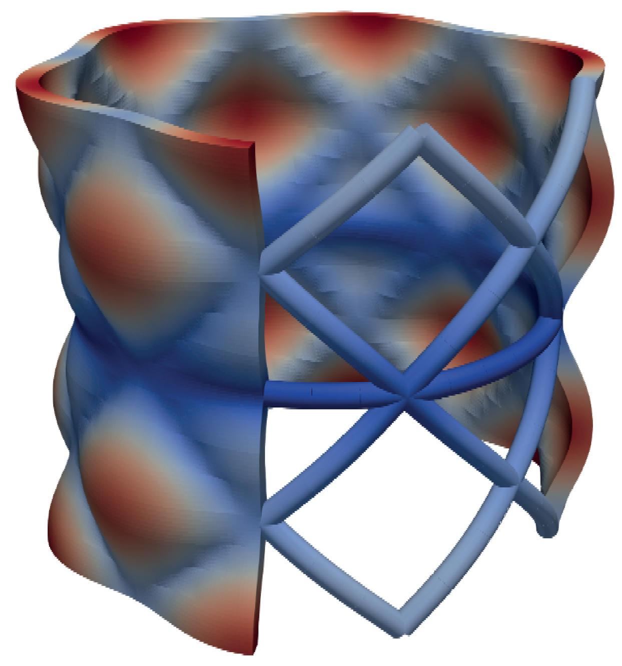


Spatial convergence is optimal for reasonable beam to solid element size ratios.

applications / validation



Composite plate
The fibers in the plate are modelled as individual beams. Comparison with a homogenized model shows excellent agreement.



Reinforced pipe under pressure
This system can not be modeled with a homogenized approach. The solid is discretized with isogeometric elements.

References

[1] Steinbrecher, I., Popp, A. (2019): Beam-to-Solid Interactions Based on a Mortar Finite Element Approach, *Paper in preparation*.

[2] Meier, C., Popp, A. and Wall, W. A (2017): Geometrically Exact Finite Element Formulations for Slender Beams: Kirchhoff–Love Theory Versus Simo–Reissner Theory. *Archives of Computational Methods in Engineering* 26(1), pp. 163–243.

[3] Puso, M. A. (2004): A 3D Mortar Method for Solid Mechanics. *International Journal for Numerical Methods in Engineering* 59(3), pp. 315–336.

[4] Farah, P., Popp, A. and Wall, W. A (2015): Segment-Based vs. Element-Based Integration for Mortar Methods in Computational Contact Mechanics. *Computational Mechanics* 55(1), pp. 209–228.